

# NAVAL POSTGRADUATE SCHOOL Monterey, California



## THESIS

### STATISTICAL ANALYSIS OF JAPANESE DEFENSE POLICY IN TRANSITION : 1983 - 1993

by

Yorihiro Nowada

September 1995

Thesis Advisor:

Katsuaki L. Terasawa

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TRANSITION : 1983 - 1993**

Yorihiro Nowada  
Lieutenant Commander, Japan Maritime Self Defense Force  
B.S., Chuo University, 1983

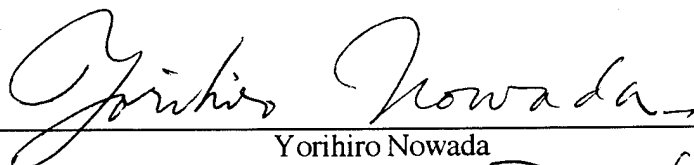
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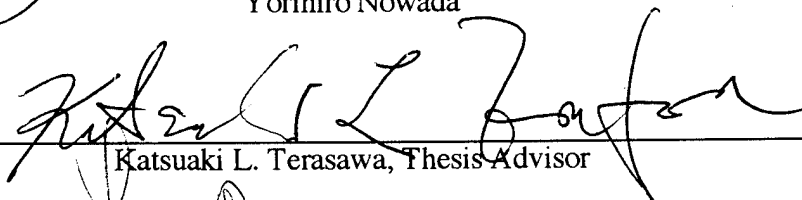
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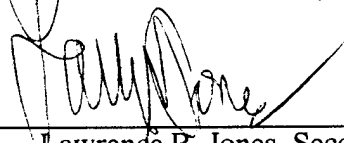
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September 1995**


Author: \_\_\_\_\_

  
Yorihiro Nowada

Approved by: \_\_\_\_\_

  
Katsuaki L. Terasawa, Thesis Advisor

  
Lawrence R. Jones, Second Reader

  
Reuben Harris, Chairman, Department of System Management



## **ABSTRACT**

This thesis examines Japanese defense policy during the transition period. It focuses on Japanese defense expenditures as a medium for comparison between Japanese and other countries' defense policies. It also selects the U.S. and the Soviet Union as influential allies and adversaries, respectively, of Japan. Using historical data of the three countries' defense expenditures, model simulations are performed. The model adopted here is the Terasawa and Gates Commitment-Based Model of Defense Allies and Adversaries.

The examination reveals that Japanese defense expenditures in 1983 ~1993 may have been based on reasons other than selective security considerations related to the U.S. and the Soviet Union. It is very clear that the U.S. and the Soviet Union impacted each other.

It is concluded that Japanese defense policy, viewed from the perspective of defense expenditures, is difficult to explain in terms of relative power-politics during the transition period.



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## I. INTRODUCTION

The severe military confrontation between the East and the West, which was called the Cold War, is over. Now there is only a remote possibility that a global-scale war may happen. However, various regional conflicts due to religious or ethnic discord are occurring, and the threat of expanding regional conflicts has increased. Countries around the world are searching for new policy which is appropriate for such new international situations.

National defense policy is affected by numerous factors both domestic and international. These factors include the nation's domestic political, economic and military conditions, and the international relationships among allies and adversaries. While domestic factors at times seem arbitrary, external factors relate to the relationships with other nations and depend on the fabric of beliefs among nations.

Former president Bush states, "The relationships that develop among allies and adversaries during the transition between the old and new world order will depend on the network of beliefs among nations concerning the commitment to cooperative or adversarial activities. These beliefs will be affected by numerous factors including the national security policy of a nation which contains political, economic and military components. The political and military components of the strategy, in turn, determine the structure of diplomatic relationships and the military posture and actions of a nation, both of which update the beliefs of others. Ultimately, a nation develops a reputation for fulfilling its explicit and implicit commitments." [Ref. 3, p. 1]

U.S. national security policy not only influenced the beliefs of other nations throughout the Cold War era, but is influential even today because of U.S. military superiority and its flexibility of response to changing situations. For example, when the U. S. considered that the international security environment had changed with the breakup of the Soviet Union and the Warsaw Pact Organization, the U.S. promptly designed a new defense policy that places priority on addressing regional threats around the world. This policy was quite different from the policy in effect during the era of Soviet power and includes the following four epoch-making principles as its military components.

- The maintenance of effective strategic deterrence
- The maintenance of forward deployment
- The ability to respond to regional and local contingencies
- Reconstitution of military forces

In consideration of financial constraints as well as the above stated principles, the U.S. defined the necessary capability to maintain security for the U.S. and its allies. The U.S. established a minimum base force which will be reconstituted under this new security environment. It is needless to say that the beliefs and defense policy of other nations, which include members of the North Atlantic Treaty Organization (NATO), Russia , Japan and so forth, were also influenced by the new U.S. policy and the changes in its military force structure.

The purpose of this thesis is to examine Japanese defense policy, which might have an impact on or be affected by other nations' defense policies, from the middle of the 1980s to the early 1990s with an emphasis on defense expenditures. To do this, Japanese defense expenditures, as well as those of the U.S. and the Soviet Union, will be simulated using a modern defense alliance model.

Following this introduction, the simulation model will be introduced in Chapter II, and the data will be provided in Chapter III. A simulation will be performed in Chapter IV, and the conclusions summarized in Chapter V.

## II. MODELING

### A. DEFENSE ALLIANCES MODELS

National defense is a classic example of a public good in economics. [Ref. 7, p. 661] Notable characteristics of a public good are nonrivalry and nonexclusiveness. Nonrivalry means that the marginal cost of providing the good to an additional consumer is zero, and nonexclusiveness means that no one can be excluded from enjoying it. National defense is a public good in that it provides both nonrivalous and nonexcludable benefits to all people and the nation.

This public good paradigm may be applied to explain defense expenditures by members of defense alliances, as well.

Terasawa and Gates state, "Alliance members benefit from their allies' defense expenditures, and it is impossible for the provider to withhold these benefits. Because of these "spill-over" benefits, national defense budgets in individual countries reflect the public benefits each country expects to receive from its allies." [Ref. 1, p. 1]

Thus, defense alliances may be modeled applying the public goods paradigm. In fact, several models have been developed to explain defense expenditures in alliances in the real world. NATO is a prime example. It was first modeled using the Pure Public Goods Model by Olson and Zeckhauser. This model showed that individuals making independent decisions provide public goods until the marginal cost of the last unit they provide equals the marginal benefit they receive from that unit. With this model, Olson and Zeckhauser concluded that suboptimality and disproportionality are inherent in defense alliances and explained NATO members' defense expenditures in 1964. [Ref. 1, p. 2]

The following model, referred to as the Joint Product Model, was introduced by Sandler and Forbes. In this model, defense expenditures provide both public and private benefits, depending on the type of weapon systems bought. They noticed that protective weapons<sup>1</sup> yield private benefits, while deterrent weapons<sup>2</sup> give public benefits. An increase in private benefits relative to public benefits reduces the incidence of attaining free defensive benefits, and should shift the defense burden from a higher GNP country to

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<sup>1</sup> Strategic nuclear weapons are examples of deterrent weapons.

<sup>2</sup> Tactical nuclear and conventional forces are examples of protective weapons.

lower ones. [Ref. 1, p. 4] Using this model, they explained the burden shift from the U.S. to Western Europe in NATO after the late 1960s.

The Joint Product Model was modified by Murdoch, Sandler, and Hansen using the concept of complementing effects between weapons. They suggest that weapons may be either substitutes or complements. They also suggest that deterrent and protective weapons among nuclear allies, and conventional weapons among non-nuclear allies, have become complementary. If weapons are complementary, alliance members might increase defense expenditures as their allies' defense expenditures increase.[Ref. 1, p. 5] They found that defense expenditures for many NATO members during the late 1970s and early 1980s were consistent with the complementary joint product model.

These are the main alliance models developed earlier in the post-WWII period. However, these models have a common feature in that they do not consider differences in levels of commitment among allies, nor interactions with adversaries. Although they can determine a nation's defense expenditure by the nation's level of GNP and the level of defense expenditures by that country's allies, they still have serious limitations in explaining defense expenditures in alliances because of their indifference to commitment and interactions. [Ref. 1, p. 1]

Considering commitment within an alliance and interactions between allies and adversaries, a Commitment-Based Model of Defense Allies and Adversaries developed by Terasawa and Gates succeeded in explaining NATO members' defense expenditures after the mid-1960s. This model distinguishes military capability from the commitment to use that capability. It also makes the model sensitive to the commitment among allies and weakens the relationships between GNP and defense expenditures. The model is tailored to effects of adversaries as well as allies because it takes into account that the total impact of any change in alliance depends on the reactions of both the other allies and potential adversaries.

In this examination of Japanese defense issues, the levels of commitment in the alliance between the U.S. and Japan, and the levels of threat perception among three countries including the Soviet Union will be considered. This is a reason for application of the Terasawa and Gates, Commitment-Based Model of Defense Allies and Adversaries in simulation.

The following section introduces this model.

## B. A COMMITMENT-BASED MODEL OF DEFENSE ALLIES AND ADVERSARIES

The Terasawa-Gates model has three countries, where each country ( $i$ ) produces a non-defense good ( $X_i$ ), and a defense good ( $Y_i$ ). The defense good is nonrivalrous in consumption. The resource constraint is given by  $G_i = X_i + P_i Y_i$ , where  $G_i$  denotes its GNP, and  $P_i$  is the relative price of the defense good measured in terms of the private good.

The utility functions of Countries One, Two and Three are represented by  $U_1 = U_1(X_1, Z_1)$ ,  $U_2 = U_2(X_2, Z_2)$ ,  $U_3 = U_3(X_3, Z_3)$ , respectively.  $Z_i$  ( $i=1, 2, 3$ ) represents Country  $i$ 's consumption of the defense good ( $Y_i$ ) and the benefits it receives from other countries ( $E_{ij}Y_j$ ):

$$(1) \quad Z_i = \sum_j E_{ij} Y_j \quad \text{for all } i \text{ and } j, \text{ where } E_{ii} = 1 \text{ and } -1 \leq E_{ij} \leq 1, \quad i \neq j.$$

Country  $j$  produces  $Y_j$  of the defense good, but Country  $i$  perceives that only  $E_{ij}$  of  $Y_j$  is relevant to Country  $i$ .  $E_{ij}Y_j$ , if positive, represents the defense commitment by Country  $j$ , and if negative, represents the defense threat from Country  $j$ .

In this model, Country One and Two are allies. Because they are allies, the countries' level of commitment ( $E_{12}$  and  $E_{21}$ ) is between zero and one. If  $E_{ij} = 1$ , defense expenditures by country  $j$  are purely public goods. Conversely, if  $E_{ij} = 0$ , defense expenditures are purely private goods. As  $E_{ij}$  varies from one to zero, private benefits become relatively more important.

Country Three is an adversary of Countries One and Two. For adverse countries, the level of threat ( $E_{13}$ ,  $E_{23}$ ,  $E_{31}$  and  $E_{32}$ ) is between zero and minus one. If a country perceives that its adversary's defense expenditures are fully committed to potential conflicts between the adversary and that country, then  $E_{ij}$  equals minus one. If adverse defense expenditures are not considered fully credible or if weapons are of inferior quality, and so on, the value of  $E_{ij}$  is between minus one and zero. [Ref. 1, pp. 9-10] Thus, an  $E_{ij}$  matrix represents the "world order" regarding defense issues. [Ref. 3, p. 9]

Country  $i$  ( $i=1,2,3$ ) maximizes its utility subject to its resource constraint, assuming a given value for the other countries' defense expenditures and commitment.

The Nash equilibrium<sup>3</sup> may be computed in this framework.

Next it is useful to illustrate a utility maximizing process, using a Cobb-Douglas type of utility function. (Detailed calculation is provided in Appendix A). The utility function for maximization is :

$$(2) \quad U_i = X_i^{A_i} Z_i^{B_i}$$

The coefficients  $A_i$  and  $B_i$  represent  $i$ 's utility elasticity of the private goods and defense goods, respectively.  $Z_i$  is :

$$Z_i = \sum_j E_{ij} Y_j \quad \text{for all } i \text{ and } j, \text{ where } E_{ii} = 1 \text{ and } -1 \leq E_{ij} \leq 1, \quad i \neq j.$$

The corresponding resource constraint is :

$$G_i = X_i + P_i Y_i$$

With these, each country's reaction function can be derived as :

$$(3) \quad Y_i = \left[ \frac{A_i}{(A_i + B_i)} \right] \left[ \left( \frac{B_i}{A_i} \right) \left( \frac{G_i}{P_i} \right) - \sum_j E_{ij} Y_j \right], \quad (i \neq j).$$

For three countries :

---

<sup>3</sup> An equilibrium in which each country selects its optimal defense expenditure while treating other countries defense expenditures are given . As a result, no country has any incentive to change its behavior.

$$(4.A) \quad Y_1 = \left[ \frac{A_1}{(A_1 + B_1)} \right] \left[ \left( \frac{B_1}{A_1} \right) \left( \frac{G_1}{P_1} \right) - E_{12}Y_2 - E_{13}Y_3 \right]$$

$$(4.B) \quad Y_2 = \left[ \frac{A_2}{(A_2 + B_2)} \right] \left[ \left( \frac{B_2}{A_2} \right) \left( \frac{G_2}{P_2} \right) - E_{21}Y_1 - E_{23}Y_3 \right]$$

$$(4.C) \quad Y_3 = \left[ \frac{A_3}{(A_3 + B_3)} \right] \left[ \left( \frac{B_3}{A_3} \right) \left( \frac{G_3}{P_3} \right) - E_{31}Y_1 - E_{32}Y_2 \right]$$

With voluntary and independently determined defense expenditures, Nash equilibrium occurs when the countries' reaction functions intersect. This is the only time where each country's expectations regarding the other countries' contributions coincide with the other countries' actual contributions.

For easier calculation to get  $Y_i$ , a matrix form is used. The reaction functions (4.A) ~ (4.C) will be transformed to :

$$(5.A) \quad Y_1 + a_1 E_{12} Y_2 + a_1 E_{13} Y_3 = b_1 \left( \frac{G_1}{P_1} \right)$$

$$(5.B) \quad Y_2 + a_2 E_{21} Y_1 + a_2 E_{23} Y_3 = b_2 \left( \frac{G_2}{P_2} \right)$$

$$(5.C) \quad Y_3 + a_3 E_{31} Y_1 + a_3 E_{32} Y_2 = b_3 \left( \frac{G_3}{P_3} \right)$$

$$\text{where } a_i = \frac{A_i}{(A_i + B_i)}, b_i = \frac{B_i}{(A_i + B_i)}.$$

The solution for the utility maximizing level of defense expenditure may be shown in matrix form as :

$$\Omega Y = b \left( \frac{G}{P} \right)$$

$$\begin{pmatrix} 1 & a_1 E_{12} & a_1 E_{13} \\ a_2 E_{21} & 1 & a_2 E_{23} \\ a_3 E_{31} & a_3 E_{32} & 1 \end{pmatrix} \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} = \begin{pmatrix} b_1 \frac{G_1}{P_1} \\ b_2 \frac{G_2}{P_2} \\ b_3 \frac{G_3}{P_3} \end{pmatrix}$$

In a more explicit form, it is given as :

$$(6.A) \quad Y_1 = \frac{1}{D} \left[ \begin{aligned} & (1 - a_2 a_3 E_{23} E_{32}) \left( \frac{G_1}{P_1} \right) b_1 + (a_1 a_3 E_{13} E_{32} - a_1 E_{12}) \left( \frac{G_2}{P_2} \right) b_2 \\ & + (a_1 a_2 E_{12} E_{23} - a_1 E_{13}) \left( \frac{G_3}{P_3} \right) b_3 \end{aligned} \right]$$

$$(6.B) \quad Y_2 = \frac{1}{D} \left[ \begin{aligned} & (a_2 a_3 E_{13} E_{23} - a_2 E_{21}) \left( \frac{G_1}{P_1} \right) b_1 + (1 - a_1 a_3 E_{13} E_{31}) \left( \frac{G_2}{P_2} \right) b_2 \\ & + (a_1 a_2 E_{13} E_{23} - a_2 E_{23}) \left( \frac{G_3}{P_3} \right) b_3 \end{aligned} \right]$$

$$(6.C) \quad Y_3 = \frac{1}{D} \left[ \begin{aligned} & (a_2 a_3 E_{21} E_{32} - a_3 E_{31}) \left( \frac{G_1}{P_1} \right) b_1 + (a_1 a_3 E_{12} E_{31} - a_3 E_{32}) \left( \frac{G_2}{P_2} \right) b_2 \\ & + (1 - a_1 a_2 E_{12} E_{21}) \left( \frac{G_3}{P_3} \right) b_3 \end{aligned} \right]$$

where  $D = 1 + a_1 a_2 a_3 (E_{12} E_{23} E_{31} + E_{13} E_{21} E_{32}) - a_1 a_3 E_{13} E_{31} - a_1 a_2 E_{12} E_{21} - a_2 a_3 E_{23} E_{32}$ .

Simulation and regression analyses will be performed with these functions using a spreadsheet technique.



### III. DATA

Data collected for this thesis includes the Gross National Product (GNP) and defense expenditures for the U.S., Japan, and the Soviet Union (and Russia<sup>4</sup>) from 1983 to 1993.

The Commitment Based Model of Defense Allies and Adversaries needs data on three countries. The three countries consist of two allies and one adversary. In this thesis, which examines Japanese defense issues, this translates to one ally and one adversary. Of course, the ally is the U.S. and the adversary is the Soviet Union and its successor Russia, since the U.S. was the closest partner of Japan and the Soviet Union its strongest threatening neighbor throughout the 1980s and the early 1990s.

Although the model requires many variables, the only variables obtainable from actual data are GNP and defense expenditures. In the following simulation, relative value of these data to GNP of the U.S. will be used.

Thus, Table 1 provides the general data table<sup>5</sup> for the three countries. Figures 1 and 2 display the trends of GNP and defense expenditures (absolute values) for each country, and Figure 3 shows the ratio of defense expenditures as a percent of GNP for each country.

---

<sup>4</sup> After the Soviet Union had dissolved in 1991, Russia inherited the Far East region in the vast Soviet Union area which adjoins Japan across the sea.

<sup>5</sup> Source: *World Military Expenditures and Arms Transfers 1993- 1994* : 1994, published by the United States Arms Control and Disarmament Agency.

Year	U.S.			Japan			Soviet / Russia		
	GNP	Defense Expenditures (DE)	DE as % of GNP	GNP	Defense Expenditures (DE)	DE as % of GNP	GNP	Defense Expenditures (DE)	DE as % of GNP
1983	4,665,000	308,800	6.62	2,932,000	28,630	0.98	2,728,000	355,100	13.02
1984	5,155,000	321,500	6.24	3,059,000	29,820	0.97	2,750,000	357,600	13.00
1985	5,304,000	337,800	6.37	3,217,000	31,390	0.98	2,771,000	362,700	13.09
1986	5,450,000	357,900	6.57	3,303,000	32,880	1.00	2,867,000	366,400	12.78
1987	5,612,000	355,800	6.34	3,446,000	34,580	1.00	2,899,000	374,200	12.91
1988	5,837,000	348,500	5.97	3,661,000	36,250	0.99	2,982,000	379,300	12.72
1989	5,993,000	346,000	5.77	3,838,000	37,700	0.98	3,010,000	344,800	11.46
1990	6,071,000	333,900	5.50	4,021,000	39,130	0.97	2,901,000	318,400	10.98
1991	6,029,000	294,400	4.88	4,193,000	40,460	0.96	2,659,000	273,100	10.27
1992	6,157,000	311,800	5.06	4,250,000	41,330	0.97	870,600	145,400	16.70
1993	6,348,000	297,600	4.69	4,260,000	41,730	0.98	777,400	113,800	14.64

Table 1. GNP and defense expenditures for three countries during 1983 ~ 1993  
(Millions of 1993 dollar)

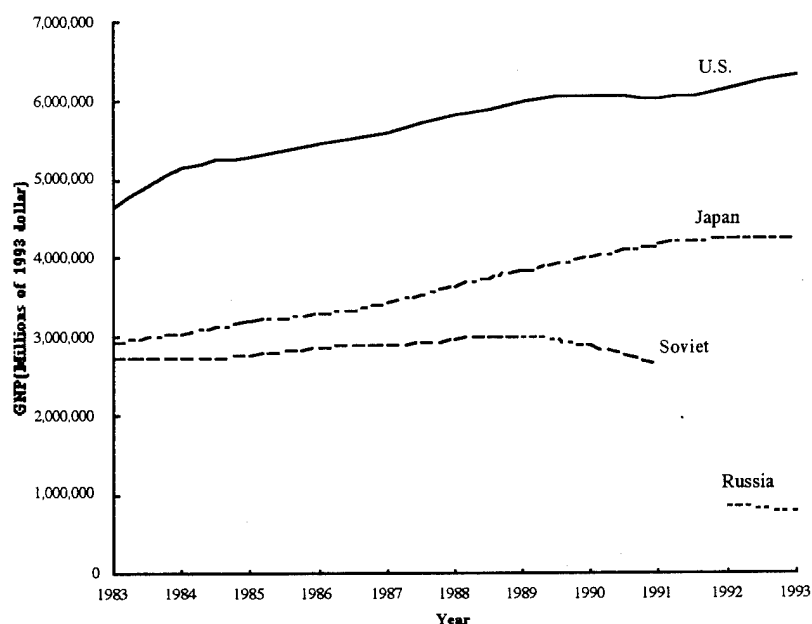


Figure 1. GNP for three countries during 1983 ~ 1993

GNP is a realistic measure of the national power of each country and provides for an excellent comparison with other countries.

In Figure 1, we can see that :

- U.S. GNP and Japanese GNP increased at a rate of 3.1 percent and 3.8 percent, respectively.
- Japanese GNP was about two third of the U.S. GNP during that time period.
- Soviet GNP was almost constant during 1983 ~ 1991.
- Accordingly, differences in the amount between the Soviet GNP and the U.S. GNP, and between the Soviet GNP and Japanese GNP, were increasing with years.
- There was quite a difference between the Soviet GNP and Russian GNP.

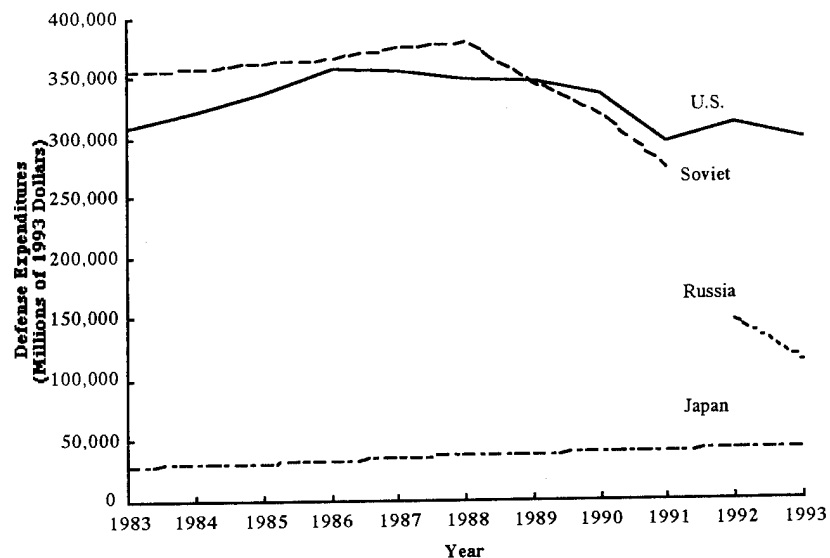


Figure 2. Defense expenditures (absolute value)  
for three countries during 1983 ~ 1993

Defense expenditures are equivalent to an amount of goods provided to defend one's own country and an ally. Although defense expenditures are interesting figures to analyze, they vary widely according to a countries' economic scales, and it is dangerous to simply compare one with another.

In Figure 2, we can see that :

- The U.S defense expenditures increased at a rate of 5 percent during 1983 ~ 1986, but decreased at a rate of 2.6 percent after that time.
- The U.S. defense expenditures surpassed the Soviet defense expenditures in 1989.
- Defense expenditures in the Soviet Union increased at a rate of 1.3 percent until 1988, but decreased at a rate of 10 percent after that time.
- Russian defense expenditures were less than half of the U.S. defense expenditures, and steadily decreased.

- On the other hand, Japanese defense expenditures increased at a rate of 3.8 percent during that time period.
- Japanese defense expenditures were about 10 percent of the U.S. defense expenditures during the 1980s, but increased to about 14 percent in the early 1990s.

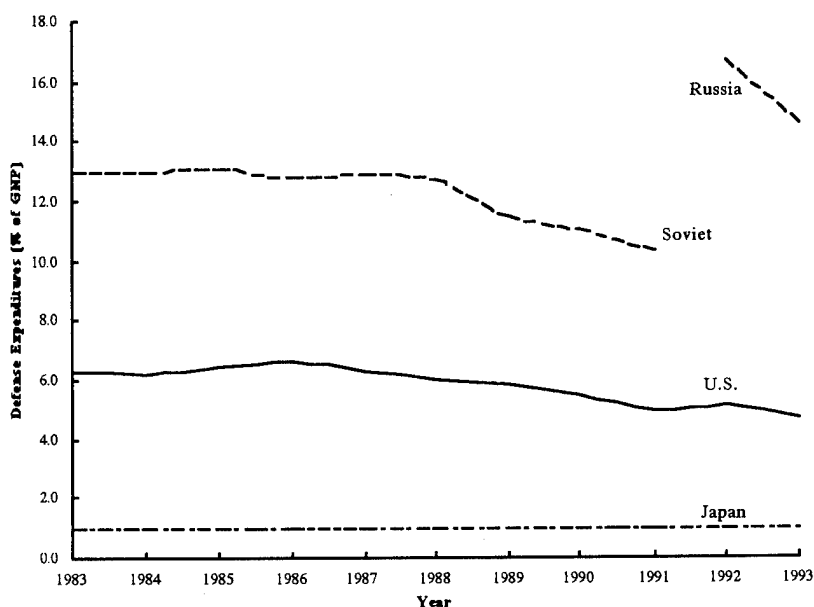


Figure 3. Defense expenditures (as % of GNP)  
for three countries during 1983 ~ 1993

The ratio of defense expenditures as a percent of GNP adds important information to that of Figure 2. With this ratio, we can measure how much GNP a country devotes to own defense and how much it shoulders defense burden in an alliance.

In Figure 3, we see that :

- Ratios of the U.S. defense expenditures as a percent of GNP decreased from 6.57 percent to 4.69 percent in seven years after 1986.

- Ratios of the Soviet defense expenditures as a percent of GNP decreased from 12.72 percent to 10.27 percent in four years after 1987.
- Ratios of Russian defense expenditures as a percent of GNP decreased from 16.7 percent to 14.64 percent during one year (1992 ~ 1993).
- On the other hand, Japanese ratios of defense expenditures were a constant 1 percent during the entire period.

## IV. SIMULATION

The purpose of this thesis includes an examination of military conditions surrounding Japan during 1983 ~ 1993. Military conditions are important factors that determined Japanese defense expenditures during that time. Factors include Japanese defense strategy, the amount of GNP and/or the situation of the U.S. and the Soviet Union. A Commitment-Based Model of Defense Allies and Adversaries will be applied using these variables, such as  $A_i, P_i$  and  $E_{ij}$ . Particularly,  $E_{ij}$  (levels of commitments between allies and perceived threat among adversaries) is a main variable which relates to military conditions. Thus, a goal in the following simulations is to derive  $E_{ij}$  for the three countries during 1983~1993.

The simulation is composed of three parts. The first one is a simulation by spreadsheet analysis and the second one is a simulation by regression analysis. These analyses will be done under the Nash equilibrium. The last one is also a simulation by regression analysis, but in this case, the Leader - Follower situation<sup>6</sup> is assumed instead of the Nash equilibrium.

Results of the simulations will be provided in the next three sections, and they will be summarized in the next chapter. The detailed processes of calculations and whole tables of results for each simulation are included as appendices.

### A. SIMULATION BY SPREADSHEET ANALYSIS

As demonstrated in the previous chapter, three countries' reaction functions for a Nash equilibrium are transformed as (6.A), (6.B) and (6.C) :

$$Y_1 = \frac{1}{D} \begin{bmatrix} (1 - a_2 a_2 E_{23} E_{32}) \left( \frac{G_1}{P_1} \right) b_1 + (a_1 a_3 E_{13} E_{32} - a_1 E_{12}) \left( \frac{G_2}{P_2} \right) b_2 \\ + (a_1 a_2 E_{12} E_{23} - a_1 E_{13}) \left( \frac{G_3}{P_3} \right) b_3 \end{bmatrix}$$

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<sup>6</sup> In the Leader - Follower situation a Leader who has all information for deciding his strategy and Followers who make a decision under the effects of the Leader's decision are assumed.

$$Y_2 = \frac{1}{D} \left[ \begin{aligned} &(a_2 a_3 E_{13} E_{23} - a_2 E_{21}) \left( \frac{G_1}{P_1} \right) b_1 + (1 - a_1 a_3 E_{13} E_{31}) \left( \frac{G_2}{P_2} \right) b_2 \\ &+ (a_1 a_2 E_{13} E_{23} - a_2 E_{23}) \left( \frac{G_3}{P_3} \right) b_3 \end{aligned} \right]$$

$$Y_3 = \frac{1}{D} \left[ \begin{aligned} &(a_2 a_3 E_{21} E_{32} - a_3 E_{31}) \left( \frac{G_1}{P_1} \right) b_1 + (a_1 a_3 E_{12} E_{31} - a_3 E_{32}) \left( \frac{G_2}{P_2} \right) b_2 \\ &+ (1 - a_1 a_2 E_{12} E_{21}) \left( \frac{G_3}{P_3} \right) b_3 \end{aligned} \right]$$

where  $a_i = \frac{A_i}{(A_i + B_i)}$ ,  $b_i = \frac{B_i}{(A_i + B_i)}$ .

Variables included in these three functions are :

- Known variables :  $G_1, G_2, G_3$  and  $Y_1^*, Y_2^*, Y_3^*$ ,
- Unknown variables :  $a_1, a_2, a_3, b_1, b_2, b_3, P_1, P_2, P_3, E_{12}, E_{13}, E_{21}, E_{23}, E_{31}, E_{32}$ .

Because there are too many unknown variables, it was necessary to set several conditions to simplify the spreadsheets. These conditions are :

- The utility elasticity of the non-defense goods for three countries are equal and fixed<sup>7</sup>. ( $A_1 = A_2 = A_3 = 0.4$ )

- The utility elasticity of defense goods for Japan is fixed. ( $B_2 = 0.1$ ) Those for the U.S. ( $B_1$ ) and the Soviet Union ( $B_3$ ) are adjusted respectively so that simulated ( $Y_i^*$ ) and actual defense expenditures ( $Y_i$ ) are equal for all three countries.

<sup>7</sup> Fixed numbers,  $A_i = 0.4$  and  $B_2 = 0.1$ , are the fittest ones gained by spreadsheet analyses.

- The relative price of defense goods measured in terms of non-defense goods for three countries is fixed<sup>8</sup>. ( $P_1=P_2=P_3=1.0$ )

- Perceived threat for the U.S.(from the Soviet Union) and for the Soviet Union (from the U.S. and Japan) are equal (not fixed) ( $E_{13}=E_{31}=E_{32}$ )<sup>9</sup>.

With these conditions and the given values of the three countries' GNPs( $G_1, G_2$  and  $G_3$ ), a simulation will be performed for every year<sup>10</sup>. An example of the spreadsheets follows :

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<sup>8</sup> It was assumed there are no differences in relative prices between defense and private goods for three countries.

<sup>9</sup> It was assumed the U.S. and the Soviet Union perceive the same level of threat with each other, and the Soviet Union recognizes that the threat from Japan is equal to the threat from the U.S., because, while Japan does not have explicit aggressive weapons, it supports the U.S. expeditionary force by all measures.

<sup>10</sup> Although the spreadsheet in this simulation still has seven variables, that is,  $E_{12}, E_{21}, E_{23}, E_{13}(=E_{31}=E_{32})$  and  $Y_1^*, Y_2^*, Y_3^*$ , simulated  $Y_1^*, Y_2^*$  and  $Y_3^*$  will be equated with actual  $Y_1, Y_2$  and  $Y_3$ . As a result, there are four variables in the model eventually.

1987

i	A <sub>i</sub>	B <sub>i</sub>	P <sub>i</sub>	G <sub>i</sub>	a <sub>i</sub>	b <sub>i</sub>	Y <sub>i</sub>	GNP	DE
1 (U.S.)	0.4	0.0103	1.0	10.0	0.97	0.03	<b>0.63</b>	5612000	355800
2 (Japan)	0.4	0.01	1.0	6.1	0.98	0.02	<b>0.06</b>	3446000	34580
3 (USSR)	0.4	0.0233	1.0	5.2	0.94	0.06	<b>0.67</b>	2899000	374200

E <sub>12</sub>	E <sub>13</sub>	E <sub>21</sub>	E <sub>23</sub>	E <sub>31</sub>	E <sub>32</sub>	D	Y <sub>1</sub> *	Y <sub>2</sub> *	Y <sub>3</sub> *
0.3	-0.6	0.0	-0.7	-0.6	-0.6	0.39	0.68	0.99	1.23
0.3	-0.6	0.1	-0.7	-0.6	-0.6	0.40	0.68	0.88	1.17
0.3	-0.6	0.2	-0.7	-0.6	-0.6	0.40	0.67	0.77	1.10
0.3	-0.6	0.3	-0.7	-0.6	-0.6	0.41	0.66	0.66	1.04
0.3	-0.6	0.4	-0.7	-0.6	-0.6	0.41	0.66	0.56	0.97
0.3	-0.6	0.5	-0.7	-0.6	-0.6	0.41	0.65	0.45	0.91
0.3	-0.6	0.6	-0.7	-0.6	-0.6	0.42	0.65	0.35	0.85
0.3	-0.6	0.7	-0.7	-0.6	-0.6	0.42	0.64	0.25	0.79
0.3	-0.6	0.8	-0.7	-0.6	-0.6	0.42	0.63	0.15	0.73
<b>0.3</b>	<b>-0.6</b>	<b>0.9</b>	<b>-0.7</b>	<b>-0.6</b>	<b>-0.6</b>	0.43	<b>0.63</b>	<b>0.06</b>	<b>0.67</b>
0.3	-0.6	1.0	-0.7	-0.6	-0.6	0.43	0.62	-0.04	0.62
0.4	-0.6	0.0	-0.8	-0.6	-0.6	0.39	0.68	0.99	1.23
0.4	-0.6	0.1	-0.8	-0.6	-0.6	0.39	0.54	1.00	1.16
0.4	-0.6	0.2	-0.8	-0.6	-0.6	0.39	0.55	0.91	1.11
0.4	-0.6	0.3	-0.8	-0.6	-0.6	0.38	0.55	0.82	1.06
0.4	-0.6	0.4	-0.8	-0.6	-0.6	0.38	0.56	0.72	1.01
0.4	-0.6	0.5	-0.8	-0.6	-0.6	0.37	0.57	0.62	0.96
0.4	-0.6	0.6	-0.8	-0.6	-0.6	0.36	0.58	0.52	0.91
0.4	-0.6	0.7	-0.8	-0.6	-0.6	0.36	0.59	0.41	0.85
0.4	-0.6	0.8	-0.8	-0.6	-0.6	0.35	0.60	0.30	0.80
0.4	-0.6	0.9	-0.8	-0.6	-0.6	0.35	0.61	0.19	0.74
0.4	-0.6	1.0	-0.8	-0.6	-0.6	0.34	0.62	0.08	0.68

Table 2. Example of Spreadsheets (An extract from Spreadsheets for 1987)

When simulated defense expenditures ( $Y_i^*$ ) are equal to actual defense expenditures ( $Y_i$ ) for all three countries ( $Y_1^*=Y_1, Y_2^*=Y_2, Y_3^*=Y_3$ ) by adjusted  $B_1$  and  $B_3$ , the simulated commitments between allies ( $E_{12}$  and  $E_{21}$ ) and perceived threats among adversaries ( $E_{23}$  and  $E_{13}=E_{31}=E_{32}$ ) will be solved to explain military conditions around Japan. In Table 2, when  $B_1 = 0.0103$  and  $B_3 = 0.0233$ , the solutions are  $E_{12}=0.3$ ,  $E_{21}=0.9$ ,  $E_{23}=-0.7$  and  $E_{13}=E_{31}=E_{32}=-0.6$  to explain  $Y_1 = 0.63$ ,  $Y_2=0.06$  and  $Y_3=0.67$  in 1987.

The definitions of  $E_{ij}$  once again are as follows :

$E_{12}$  : The level of Japanese commitment which the U.S. perceived.

$E_{13}$  : The level of the Soviet threat which the U.S. perceived.

$E_{21}$  : The level of the U.S. commitment which Japan perceived.

$E_{23}$  : The level of the Soviet threat which Japan perceived.

$E_{31}$  : The level of the U.S. threat which the Soviet Union perceived.

$E_{32}$  : The level of Japanese threat which the Soviet Union perceived.

These definitions of  $E_{ij}$  may be easily displayed in a table.

	U.S.	Japan	U.S.S.R
U.S.	—	$E_{12}$	$E_{13}$
Japan	$E_{21}$	—	$E_{23}$
U.S.S.R	$E_{31}$	$E_{32}$	—

Table 3. Combinations of  $E_{ij}$

Moreover, the interpretations of the sign(+,-) and value of  $E_{ij}$  are as follows :

- $E_{ij}$  between zero and one ( $0 \leq E_{ij} \leq 1$ ) means a commitment in an alliance, and the greater  $E_{ij}$  is, the greater the level of commitment becomes.
- $E_{ij}$  between minus one and zero ( $-1 \leq E_{ij} \leq 0$ ) means a threat with adversaries, and the greater negative value  $E_{ij}$  is, the greater the level of threat.

Thus, the solutions of Table 2 may be expressed in a table as follows :

	U.S.	Japan	U.S.S.R
U.S.	—	0.3	-0.6
Japan	0.9	—	-0.7
U.S.S.R	-0.6	-0.6	—

Table 4. Example of solutions in the  $E_{ij}$  table

Table 4 shows that in 1987 :

- The U.S. perceived the Japanese commitment as 0.3.
- The U.S. perceived the Soviet threat as -0.6.
- Japan perceived the U.S. commitment as 0.9.
- Japan perceived the Soviet threat as -0.7.
- The Soviet Union perceived the U.S. threat as -0.6.
- The Soviet Union perceived the Japanese threat as -0.6.

All solutions of the simulation during 1983 ~ 1993 are as follows :

	$E_{21}$	$E_{23}$	$E_{12}$	$E_{32}=E_{13}=E_{31}$
1983	0.6	-0.4	0.4	-0.6
1984	0.8	-0.6	0.4	-0.6
1985	0.9	-0.7	0.3	-0.6
1986	0.9	-0.7	0.3	-0.6
1987	0.9	-0.7	0.3	-0.6
1988	0.8	-0.6	0.3	-0.6
1989	0.4	-0.2	0.3	-0.6
1990	0.4	-0.2	0.3	-0.6
1991	0.3	-0.1	0.3	-0.6
1992	0.4	-0.4	0.3	-0.4
1993	0.4	-0.5	0.3	-0.3

Table 5. Simulated  $E_{ij}$  during 1983 ~1993

Finally, Figure 4 provides overall trends of  $E_{ij}$ .

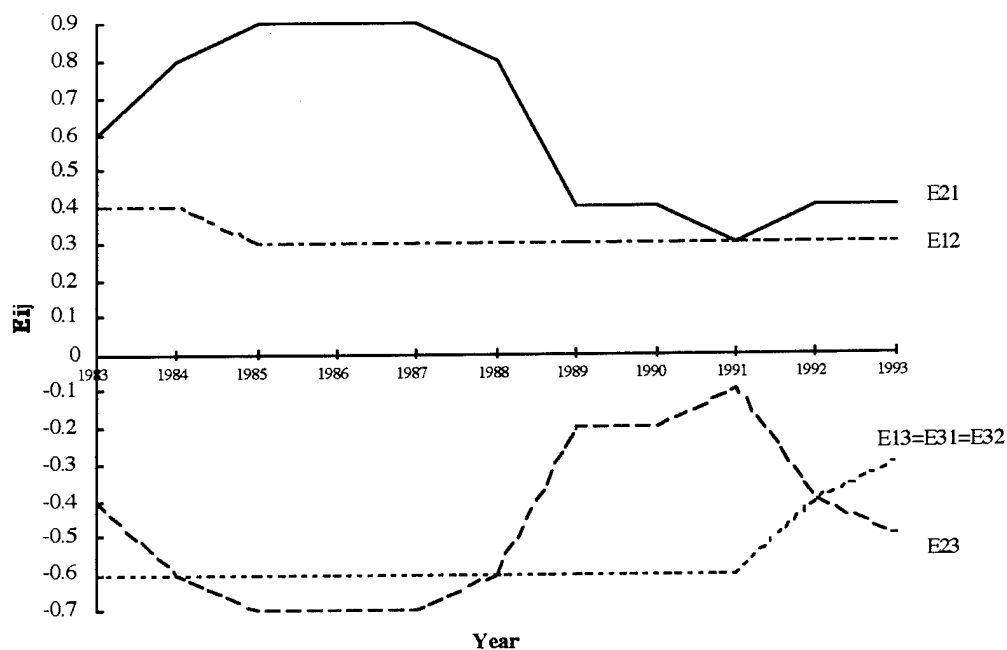


Figure 4. Trends of  $E_{ij}$  during 1983 ~ 1993

Based on an analysis of these results, the following three points may be made :

- a. There was a reduction in adversarial intensity between the U.S. and the Soviet Union/ Russia post-1991( $E_{13}=E_{31}=E_{32}$  negative value decreased.). This reduction was related to the events leading to the collapse of the Soviet Union in 1991.
- b. There was a reduction in how the Japanese perceived the U.S. commitments and the Soviet / Russian threats post-1987 ( $E_{21}$  and  $E_{23}$  went down.). These reductions resulted from a decrease in Soviet defense expenditures (absolute value) after 1988 and a reversal of defense expenditures between the Soviet Union and the U.S. after 1989.

c. There was a reemergence of Japanese perceived threat from Russia post-1991( $E_{23}$  negative value rose.). This was caused in great part by a reduction in adversarial intensity between the U.S. and the Soviet Union/Russia.



## B. SIMULATION BY REGRESSION ANALYSIS (A)

The spreadsheet analysis completed in the previous section has limitations in explaining defense expenditures because of its a priori restrictions on  $A_i$ ,  $B_i$ ,  $P_i$  and  $E_{ij}$ . Although these restrictions are not unreasonable and  $E_{ij}$ 's are selected to be consistent with the data, the simulation results cannot be used without sufficient consideration of the limitations. Particularly, the conditions for  $B_i$  and  $E_{ij}$  in that  $B_1$  and  $B_3$  are adjusted so that simulated  $Y_i^*$  and actual  $Y_i$  are equal for all three countries while  $B_2$  is fixed. The perceptions of threats except for Japan ( $E_{23}$ ), are equated with one another ( $E_{13}=E_{31}=E_{32}$ ), which can seriously affect the results.

Regression analysis will be applied to reduce the limitations by freeing, not fixing, all the parameters. However, it cannot be helped that some values in the reaction functions for the Nash equilibrium are approximated in the process of making a regression model using this method.

The process of applying a regression model is provided below.

The transformed three countries' reaction functions are again listed as (6.A), (6.B) and (6.C) :

$$Y_1 = \frac{1}{D} \left[ \begin{array}{l} (1 - a_2 a_2 E_{23} E_{32}) \left( \frac{G_1}{P_1} \right) b_1 + (a_1 a_3 E_{13} E_{32} - a_1 E_{12}) \left( \frac{G_2}{P_2} \right) b_2 \\ + (a_1 a_2 E_{12} E_{23} - a_1 E_{13}) \left( \frac{G_3}{P_3} \right) b_3 \end{array} \right]$$

$$Y_2 = \frac{1}{D} \left[ \begin{array}{l} (a_2 a_3 E_{13} E_{23} - a_2 E_{21}) \left( \frac{G_1}{P_1} \right) b_1 + (1 - a_1 a_3 E_{13} E_{31}) \left( \frac{G_2}{P_2} \right) b_2 \\ + (a_1 a_2 E_{13} E_{23} - a_2 E_{23}) \left( \frac{G_3}{P_3} \right) b_3 \end{array} \right]$$

$$Y_3 = \frac{1}{D} \left[ \begin{array}{l} (a_2 a_3 E_{21} E_{32} - a_3 E_{31}) \left( \frac{G_1}{P_1} \right) b_1 + (a_1 a_3 E_{12} E_{31} - a_3 E_{32}) \left( \frac{G_2}{P_2} \right) b_2 \\ + (1 - a_1 a_2 E_{12} E_{21}) \left( \frac{G_3}{P_3} \right) b_3 \end{array} \right]$$

where  $D = 1 + a_1 a_2 a_3 (E_{12} E_{23} E_{31} + E_{13} E_{21} E_{32}) - a_1 a_3 E_{13} E_{31} - a_1 a_2 E_{12} E_{21} - a_2 a_3 E_{23} E_{32}$

Approximation for the regression model is defined as follows :

- From the definitions of a Cobb-Douglas function and  $E_{ij}$ , absolute values of all  $a_i$ ,  $b_i$  and  $E_{ij}$  are less than zero. Therefore,  $a_i a_j E_{ij} E_{ij}$  and  $a_i a_j a_k E_{ij} E_{ij} E_{ij}$  in the above functions become very small and are approximate zero.

With this approximation, the functions (6.A), (6.B) and (6.C) are transformed to three regression models as follows :

$$(7.A) \quad Y_1 = (b_1) \frac{G_1}{P_1} - (b_2 a_1 E_{12}) \frac{G_2}{P_2} - (b_3 a_1 E_{13}) \frac{G_3}{P_3}$$

$$(7.B) \quad Y_2 = (b_2) \frac{G_2}{P_2} - (b_1 a_2 E_{21}) \frac{G_1}{P_1} - (b_3 a_2 E_{23}) \frac{G_3}{P_3}$$

$$(7.C) \quad Y_3 = (b_3) \frac{G_3}{P_3} - (b_1 a_3 E_{31}) \frac{G_1}{P_1} - (b_2 a_3 E_{32}) \frac{G_2}{P_2}$$

Because  $G_i$  and  $Y_i$  are given by actual data , three multiple regression models are prepared with three independent variables (one  $b_i$  and two  $b_i a_j E_{ij}$ ) respectively.  $P_i$  assumes a value dependent on the particular country.

Table 6 provides data for the regression analysis<sup>11</sup> and Table 7 shows output from the three regression models (7.A), (7.B) and (7.C)<sup>12</sup>.

<sup>11</sup> In Table 6., each series of data is smoothed by the moving average method.

<sup>12</sup>  $P_1$ ,  $P_2$  and  $P_3$  are assumed to be one.

Year	$G_1/P_1$	$G_2/P_2$	$G_3/P_3$	$Y_1$	$Y_2$	$Y_3$
1983	10.00	6.1096	5.5912	0.6428	0.0596	0.7274
1984	10.00	6.0948	5.4689	0.6408	0.0595	0.7129
1985	10.00	6.0199	5.2732	0.6391	0.0591	0.6833
1986	10.00	6.0887	5.2169	0.6425	0.0604	0.6743
1987	10.00	6.1577	5.1784	0.6292	0.0614	0.6630
1988	10.00	6.2722	5.0990	0.6028	0.0622	0.6306
1989	10.00	6.4332	4.9699	0.5748	0.0632	0.5832
1990	10.00	6.6607	4.7371	0.5385	0.0648	0.5176
1991	10.00	6.8269	3.5343	0.5149	0.0662	0.4045
1992	10.00	6.8561	2.3497	0.4878	0.0667	0.2895
1993	10.00	6.8067	1.3193	0.4876	0.0664	0.2077

Table 6. Data of ( $G_i / P_i$ ) and  $Y_i$  during 1983 ~ 1993  
( $P_1 = P_2 = P_3 = 1$ )

**(7.A)**

Multiple R	0.996					
R Square	0.992					
Adjusted R Square	0.865					
Standard Error	0.006					
	df	SS	MS	F	Significance F	
Regression	3	0.041	0.014	330	7E-08	
Residual	8	0.000	4E-05			
Total	11	0.041				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
<b>b<sub>1</sub></b>	0.1536	0.008	18.66	7E-08	0.135	0.173
<b>b<sub>2</sub>a<sub>1</sub>E<sub>12</sub></b>	-0.1558	0.011	-13.82	7E-07	-0.182	-0.130
<b>b<sub>3</sub>a<sub>1</sub>E<sub>13</sub></b>	0.0094	0.003	3.532	8E-03	0.003	0.016

**(7.B)**

Multiple R	0.989					
R Square	0.978					
Adjusted R Square	0.848					
Standard Error	0.000					
	df	SS	MS	F	Significance F	
Regression	3	8E-05	3E-05	119	2E-06	
Residual	8	2E-06	2E-07			
Total	11	9E-05				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
<b>b<sub>1</sub>a<sub>2</sub>E<sub>21</sub></b>	0.0009	0.001	1.455	0.184	-0.001	0.002
<b>b<sub>2</sub></b>	0.0084	0.001	9.804	1E-05	0.006	0.010
<b>b<sub>3</sub>a<sub>2</sub>E<sub>23</sub></b>	-0.0001	0.000	-0.395	0.703	-0.001	0.000

**(7.C)**

Multiple R	0.998					
R Square	0.996					
Adjusted R Square	0.871					
Standard Error	0.012					
	df	SS	MS	F	Significance F	
Regression	3	0.318	0.106	748	4E-09	
Residual	8	0.001	0.000			
Total	11	0.319				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
<b>b<sub>1</sub>a<sub>3</sub>E<sub>31</sub></b>	0.1416	0.015	9.262	2E-05	0.106	0.177
<b>b<sub>2</sub>a<sub>3</sub>E<sub>32</sub></b>	-0.1938	0.021	-9.254	2E-05	-0.242	-0.146
<b>b<sub>3</sub></b>	0.0850	0.005	17.15	1E-07	0.074	0.096

Table 7. Output of the original condition (A)

I calculate  $a_i$ ,  $b_i$  and  $E_{ij}$  from output data in Table 7.

$$Y_1 : \quad b_1 = 0.1536, \quad b_2 a_1 E_{12} = 0.1558 \quad \Rightarrow \quad E_{12} = 21.8413$$

$$a_1^{13} = 0.8464, \quad b_3 a_1 E_{13} = -0.0094 \quad \Rightarrow \quad E_{13} = -0.1310$$

$$Y_2 : \quad b_2 = 0.0084, \quad b_1 a_2 E_{21} = -0.0009 \quad \Rightarrow \quad E_{21} = -0.0060$$

$$a_2 = 0.9916, \quad b_3 a_2 E_{23} = 0.0001 \quad \Rightarrow \quad E_{23} = 0.0010$$

$$Y_3 : \quad b_3 = 0.0850, \quad b_1 a_3 E_{31} = -0.1416 \quad \Rightarrow \quad E_{31} = -1.0076$$

$$a_3 = 0.9150, \quad b_2 a_3 E_{32} = 0.1938 \quad \Rightarrow \quad E_{32} = 25.1270$$

These  $E_{ij}$  are inserted into the  $E_{ij}$  table.

	U.S.	Japan	U.S.S.R
U.S.	—	(21.8413)	-0.1310
Japan	(-0.0060)	—	(0.0010)
U.S.S.R	-1.0076	(25.1270)	—

Table 8.  $E_{ij}$  in Regression (A1)

Before analyzing calculated  $E_{ij}$ , the output data in Table 7 may be tested using a Statistical Fitness approach.

F-Test is a test to determine whether there is a regression relationship between  $Y_i$  and the set of  $(G_i / P_i)$  in each model by using the F distribution.  $t$ -Test examines whether the relationship between  $Y_i$  and individual regression coefficients is significant by using the  $t$  distribution. Assuming  $\alpha$  risk is 0.05, decision rules of both tests are :

<sup>13</sup> From the definitions of  $a_i$  and  $b_i$  (see the equations (6.A) ~ (6.C)), I can get an equation  $a_i = 1 - b_i$ .

- F-Test : If  $F > 4.07$ , a regression relationship exists<sup>14</sup>.
- $t$ -Test : If  $|t| > 2.306$ , a coefficient is fit<sup>15</sup>.

With these decision rules, the output of Table 7 may be interpreted as follows :

- Because all  $F$  are greater than 4.07, there is a regression relationship between  $Y_i$  and the set of  $(G_i / P_i)$  in each model,
- Because all  $t$  statistics except those for  $b_1a_2E_{21}$  and  $b_3a_2E_{23}$  are greater than 2.306, there is a significant regression relationship between  $Y_i$  and individual regression coefficients in each model except coefficients of  $b_1a_2E_{21}$  ( $=0.0009$ ) and  $b_3a_2E_{23}$  ( $=-0.0001$ ). That is, (0.0009) and (-0.0001) are unreliable coefficients statistically.

Thus, four results from the simulation are :

- $E_{21}$  and  $E_{23}$  should be excluded from further analyses because related coefficients are unreliable,
- A fit value for the model is only  $E_{13}$ ,
- $E_{12}$  and  $E_{31}$  values are acceptable in terms of sign (+ or -) . but the magnitude is problematic because the absolute value of  $E_{ij}$  should be less than or equal to one.
- Although  $t$ -statistics is satisfactory, a sign of  $E_{32}$  is wrong and the reverse of the one in the original model. It should be negative to reflect the adversarial relation between the two countries.

<sup>14</sup> From  $F$  table,  $F(0.95,3,8)=4.07$ . Calculated  $F=330.4, 118.8$  and  $748.2 > 4.07$ .

<sup>15</sup> From  $t$  table,  $T(0.975,8)=2.306$ . Calculated  $|t|=18.7, 13.8, 3.5, 9.8, 9.3, 9.3$  and  $17.1 > 2.306$ , but  $1.45$  and  $0.39 < 2.306$ .

Additionally, other regression runs were conducted using different values for  $Y_i$  and  $P_i$  <sup>16</sup>. If only a part of the U.S. and the Soviet defense expenditures affect military circumstances around Japan, (50 percent, for example), then the original  $Y_1$  and  $Y_3$  should be multiplied by 0.5. One might argue that the relative prices are different among three countries, reflecting the different efficiency of their defense industries vis à vis the civilian sector. For example, the Soviet Union might be more efficient in producing military goods than civilian goods while Japan might be more efficient in producing civilian goods than military goods. In such an example, the original  $P_3$  could be reduced while  $P_2$  could be increased.

Table 9 shows an regression output with these new assumptions on  $Y_i$  and  $P_i$  <sup>17</sup>.

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<sup>16</sup> Because these changes are proportional among data series, they don't affect F and  $t$  statistics at all.

<sup>17</sup> The less the part of the U.S. and the Soviet defense expenditures affect, the more  $E_{12}$  and  $E_{31}$  approach a fit value. In this case a fifteen percent reduction is taken from the data of William W. Kaufman. [Ref 6.] On the other hand, because the change in  $P_2$  and  $P_3$  values of  $E_{ij}$  not hardly affect values of  $E_{ij}$ ,  $P_i$  are kept equal to one.

## (7.A)

Multiple R	0.996					
R Square	0.992					
Adjusted R Square	0.865					
Standard Error	0.001					
	df	SS	MS	F	Significance F	
Regression	3	0.001	0.000	330	7E-08	
Residual	8	7E-06	9E-07			
Total	11	0.001				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
$b_1$	0.0230	0.001	18.66	7E-08	0.020	0.026
$b_2 a_1 E_{12}$	-0.0234	0.002	-13.82	7E-07	-0.027	-0.019
$b_3 a_1 E_{13}$	0.0014	0.000	3.532	0.008	0.000	0.002

## (7.B)

Multiple R	0.989					
R Square	0.978					
Adjusted R Square	0.848					
Standard Error	0.000					
	df	SS	MS	F	Significance F	
Regression	3	8E-05	3E-05	119	2E-06	
Residual	8	2E-06	2E-07			
Total	11	9E-05				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
$b_1 a_2 E_{21}$	0.0009	0.001	1.455	0.184	-0.001	0.002
$b_2$	0.0084	0.001	9.804	1E-05	0.006	0.010
$b_3 a_2 E_{23}$	-8E-05	0.000	-0.395	0.703	-0.001	0.000

## (7.C)

Multiple R	0.998					
R Square	0.996					
Adjusted R Square	0.871					
Standard Error	0.002					
	df	SS	MS	F	Significance F	
Regression	3	0.007	0.002	748	4E-09	
Residual	8	3E-05	3E-06			
Total	11	0.007				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
$b_1 a_3 E_{31}$	0.0212	0.002	9.262	2E-05	0.016	0.027
$b_2 a_3 E_{32}$	-0.0291	0.003	-9.254	2E-05	-0.036	-0.022
$b_3$	0.0128	0.001	17.15	1E-07	0.011	0.014

Table 9. Output of the fittest condition (A)

Calculated  $E_{ij}$  are :

$$Y_1 : b_1 = 0.0230, b_2 a_1 E_{12} = 0.0234 \Rightarrow E_{12} = 2.8382$$

$$a_1 = 0.9770, b_3 a_1 E_{13} = -0.0014 \Rightarrow E_{13} = -0.1135$$

$$Y_2 : b_2 = 0.0084, b_1 a_2 E_{21} = -0.0009 \Rightarrow E_{21} = -0.0400$$

$$a_2 = 0.9916, b_3 a_2 E_{23} = 0.0001 \Rightarrow E_{23} = 0.0064$$

$$Y_3 : b_3 = 0.0128, b_1 a_3 E_{31} = -0.0212 \Rightarrow E_{31} = -0.9338$$

$$a_3 = 0.9872, b_2 a_3 E_{32} = 0.0291 \Rightarrow E_{32} = 3.4932$$

In the  $E_{ij}$  table :

	U.S.	Japan	U.S.S.R
U.S.	—	(2.83823)	<b>-0.11349</b>
Japan	(-0.03997)	—	(0.00636)
U.S.S.R	<b>-0.93382</b>	(3.49318)	—

Table 10.  $E_{ij}$  in Regression (A2)

Because statistical tests remain the same as those in the first case,  $E_{21}$  and  $E_{23}$  are still unreliable, and the sign of  $E_{32}$  is unsatisfactory. Both  $E_{13}$  and  $E_{31}$  fall within the acceptable range of the values.

Although conditions for the regression analyses were varied, the commitment coefficients involving Japan ( $E_{21}$ ,  $E_{23}$ ,  $E_{12}$ ,  $E_{32}$ ) remain outside the acceptable range of values. The level of Japanese defense expenditures in 1983 - 1993 period are not explained by the interactive commitment model involving adversaries. However, the simulation of regression models shows that there is quite a disparity in the levels of perceived threat

between the U.S. and the Soviet Union / Russia. The U.S. perception of Russian threat is much smaller than that of Russia toward the U.S.

### C. SIMULATION BY REGRESSION ANALYSIS (B)

The two previous simulations were done using the Nash equilibrium. The third simulation in this thesis will be performed using a Leader - Follower framework. In a Leader - Follower framework, a Leader estimates the reaction functions of follower countries and decides his strategy, and Followers make a decision based on the Leaders' decision. While each country can do its best given what the other countries do in a Nash equilibrium case, a Leader can do his best with information of the world and the Followers can only make a choice based on the influence of the Leader. In the real world where all nations never have an equal ability in economics, diplomacy and military leadership, a Leader - Follower model probably is more applicable than a Nash equilibrium.

It seems quite natural that in a relationship between the U.S. and Japan, the U.S. is a Leader and Japan is a Follower because of their political and economic relationships.

Thus, the simulation using regression models in the prior section will be modified from a Nash equilibrium case to a Leader - Follower situation.

Assuming Country One is a Leader and Countries Two and Three are Followers, the equations (4.A) and (4.B) provide the reaction functions for Followers before transforming to the matrix form as :

$$Y_2 = \left[ \frac{A_2}{(A_2 + B_2)} \right] \left[ \left( \frac{B_2}{A_2} \right) \left( \frac{G_2}{P_2} \right) - E_{21}Y_1 - E_{23}Y_3 \right]$$
$$Y_3 = \left[ \frac{A_3}{(A_3 + B_3)} \right] \left[ \left( \frac{B_3}{A_3} \right) \left( \frac{G_3}{P_3} \right) - E_{31}Y_1 - E_{32}Y_2 \right]$$

Using  $a_i$  and  $b_i$  instead of  $A_i$  and  $B_i$  :

$$(8.A) \quad Y_2 = (b_2) \left( \frac{G_2}{P_2} \right) - (a_2 E_{21})Y_1 - (a_2 E_{23})Y_3$$

$$(8.B) \quad Y_3 = (b_3) \left( \frac{G_3}{P_3} \right) - (a_3 E_{31})Y_1 - (a_3 E_{32})Y_2$$

$$\text{where } a_i = \frac{A_i}{(A_i + B_i)}, b_i = \frac{B_i}{(A_i + B_i)}.$$

By the substitution between the equations (8.B) and (8.C), reaction functions for Followers are expressed in terms of  $Y_1$ .

$$(9.A) \quad Y_2 = \frac{(b_2) \frac{G_2}{P_2} - (a_2 E_2 - a_2 a_3 E_{23} E_{311}) Y_1 - (a_2 b_3 E_{23}) \frac{G_3}{P_3}}{1 - a_2 a_3 E_{23} E_{32}}$$

$$(9.B) \quad Y_3 = \frac{(b_3) \frac{G_3}{P_3} - (a_3 E_{31} - a_2 a_3 E_{21} E_{32}) Y_1 - (a_3 b_2 E_{32}) \frac{G_2}{P_2}}{1 - a_2 a_3 E_{23} E_{32}}$$

On the other hand, a function for a Leader is obtained by the maximization of the original utility function (2). The utility function and the resource constraint for a Leader are as follows :

$$(10) \quad U_1 = X_1^{A_1} Z_1^{B_1}$$

$$\text{subject to} \quad G_1 - X_1 - P_1 Y_1 = 0$$

$$\text{where} \quad Z_1 = Y_1 + E_{12} Y_2 + E_{13} Y_3$$

Using the Lagrangean function and the Followers' reaction functions (9.A) and (9.B), produces the defense expenditure for a Leader.

$$(11) \quad Y_1 = (b_1) \frac{G_1}{P_1} - \frac{(b_1) \left[ (A_1 b_2 E_{12} - A_1 a_3 b_2 E_{13} E_{32}) \frac{G_2}{P_2} + (A_1 b_3 E_{13} - A_1 a_2 b_3 E_{12} E_{23}) \frac{G_3}{P_3} \right]}{\begin{pmatrix} B_1 - B_1 a_2 a_3 E_{23} E_{32} - B_1 a_2 E_{12} E_{21} + B_1 a_2 a_3 E_{12} E_{23} E_{31} \\ -B_1 a_3 E_{13} E_{31} + B_1 a_2 a_3 E_{13} E_{21} E_{32} \end{pmatrix}}$$

The functions for the Leader and Followers, (9.A), (9.B) and (11), are transformed into the regression models for the three countries by the similar approximation used in the Nash case.

$$(12.A) \quad Y_1 = (b_1) \frac{G_1}{P_1} - (b_2 a_1 E_{12}) \frac{G_2}{P_2} - (b_3 a_1 E_{13}) \frac{G_3}{P_3}$$

$$(12.B) \quad Y_2 = (b_2) \frac{G_2}{P_2} - (a_2 E_{21}) Y_1 - (b_3 a_2 E_{23}) \frac{G_3}{P_3}$$

$$(12.C) \quad Y_3 = (b_3) \frac{G_3}{P_3} - (a_3 E_{31}) Y_1 - (b_2 a_3 E_{32}) \frac{G_2}{P_2}$$

Because of a Leader - Follower case, the Leader's model (12.A) is expressed by three countries' GNP data ( $G_i / P_i$ ), while the Followers' models, (12.B) and (12.C), have data on the Leader's defense expenditure ( $Y_1$ ) instead of GNP. It is interesting that, by the approximation, the parameters for the Leader's model ( $b_i, a_i, E_{ij}$ ) become the same as those for the Nash case (7.A).

In simulation, various conditions from the previous regression analysis were used. Additionally, it was assumed that the U.S. is the leader, and Japan and the Soviet Union / Russia are the followers. Results of the two cases are provided for comparison with the results of the Nash equilibrium.

An output with the original condition<sup>18</sup> is as follows :

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<sup>18</sup>  $Y_1$  and  $Y_3$  don't change and  $P_i$ s are equal to one.

**(12.A)**

Multiple R	0.996					
R Square	0.992					
Adjusted R Square	0.865					
Standard Error	0.006					
	df	SS	MS	<b>F</b>	Significance F	
Regression	3	0.041	0.014	330	7E-08	
Residual	8	0.000	4E-05			
Total	11	0.041				
	<b>Coefficients</b>	Standard Error	<b>t Stat</b>	P-value	Lower 95%	Upper 95%
<b>b<sub>1</sub></b>	0.15364	0.008	18.7	7E-08	0.135	0.173
<b>b<sub>2</sub>a<sub>1</sub>E<sub>12</sub></b>	-0.15584	0.011	-13.8	7E-07	-0.182	-0.130
<b>b<sub>3</sub>a<sub>1</sub>E<sub>13</sub></b>	0.00943	0.003	3.532	0.008	0.003	0.016

**(12.B)**

Multiple R	0.989					
R Square	0.978					
Adjusted R Square	0.847					
Standard Error	0.000					
	df	SS	MS	<b>F</b>	Significance F	
Regression	3	8E-05	3E-05	117	2E-06	
Residual	8	2E-06	2E-07			
Total	11	9E-05				
	<b>Coefficients</b>	Standard Error	<b>t Stat</b>	P-value	Lower 95%	Upper 95%
<b>b<sub>2</sub></b>	0.0094	2E-04	40.62	1E-10	0.009	0.010
<b>b<sub>3</sub>a<sub>2</sub>E<sub>23</sub></b>	-0.0001	2E-04	-0.510	0.624	-0.001	4E-04
<b>a<sub>2</sub>E<sub>21</sub></b>	0.0057	0.004	1.389	0.202	-0.004	0.015

**(12.C)**

Multiple R	0.999					
R Square	0.998					
Adjusted R Square	0.873					
Standard Error	0.008					
	df	SS	MS	<b>F</b>	Significance F	
Regression	3	0.318	0.106	1498	3E-10	
Residual	8	0.001	7E-05			
Total	11	0.319				
	<b>Coefficients</b>	Standard Error	<b>t Stat</b>	P-value	Lower 95%	Upper 95%
<b>b<sub>2</sub>a<sub>3</sub>E<sub>32</sub></b>	-0.0507	0.004	-12.9	1E-06	-0.060	-0.042
<b>b<sub>3</sub></b>	0.0758	0.004	18.6	7E-08	0.066	0.085
<b>a<sub>3</sub>E<sub>31</sub></b>	0.9325	0.070	13.4	9E-07	0.772	1.093

Table 11. Output of the original condition (B)

Calculated  $E_{ij}$  are :

$$Y_1 : \quad b_1 = 0.1536, \quad b_2 a_1 E_{12} = 0.1558 \quad \Rightarrow \quad E_{12} = 19.6471$$

$$a_1 = 0.8464, \quad b_3 a_1 E_{13} = -0.0094 \quad \Rightarrow \quad E_{13} = -0.1470$$

$$Y_2 : \quad b_2 = 0.0094, \quad a_2 E_{21} = -0.0057 \quad \Rightarrow \quad E_{21} = -0.0057$$

$$a_2 = 0.9906, \quad b_3 a_2 E_{23} = 0.0001 \quad \Rightarrow \quad E_{23} = 0.0016$$

$$Y_3 : \quad b_3 = 0.0758, \quad a_3 E_{31} = -0.9325 \quad \Rightarrow \quad E_{31} = -1.0089$$

$$a_3 = 0.9242, \quad b_2 a_3 E_{32} = 0.0507 \quad \Rightarrow \quad E_{32} = 5.8557$$

	U.S.	Japan	U.S.S.R
U.S.	—	(19.6471)	-0.1470
Japan	(-0.0057)	—	(0.0016)
U.S.S.R	-1.0089	(5.8557)	—

Table 12.  $E_{ij}$  in Regression (B1)

The fittest condition in a Leader - Follower situation is the fifteen percent reduction of  $Y_1$  and  $Y_3$ <sup>19</sup>, the same as the Nash equilibrium. An output is :

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<sup>19</sup>  $P_i$ s are still equal to one.

**(12.A)**

Multiple R	0.996					
R Square	0.992					
Adjusted R Square	0.865					
Standard Error	0.001					
	df	SS	MS	<b>F</b>	Significance F	
Regression	3	0.001	3E-04	330	7E-08	
Residual	8	7E-06	9E-07			
Total	11	0.001				
	<b>Coefficients</b>	Standard Error	<b>t Stat</b>	P-value	Lower 95%	Upper 95%
<b>b<sub>1</sub></b>	0.0230	0.001	18.7	7E-08	0.020	0.026
<b>b<sub>2</sub>a<sub>1</sub>E<sub>12</sub></b>	-0.0234	0.002	-13.8	7E-07	-0.027	-0.019
<b>b<sub>3</sub>a<sub>1</sub>E<sub>13</sub></b>	0.0014	4E-04	3.532	0.008	5E-04	0.002

**(12.B)**

Multiple R	0.989					
R Square	0.978					
Adjusted R Square	0.847					
Standard Error	0.000					
	df	SS	MS	<b>F</b>	Significance F	
Regression	3	8E-05	3E-05	117	2E-06	
Residual	8	2E-06	2E-07			
Total	11	9E-05				
	<b>Coefficients</b>	Standard Error	<b>t Stat</b>	P-value	Lower 95%	Upper 95%
<b>b<sub>2</sub></b>	0.0094	2E-04	40.62	1E-10	0.009	0.010
<b>b<sub>3</sub>a<sub>2</sub>E<sub>23</sub></b>	-0.0001	2E-04	-0.510	0.624	-0.001	4E-04
<b>a<sub>2</sub>E<sub>21</sub></b>	0.0378	0.027	1.389	0.202	-0.025	0.100

**(12.C)**

Multiple R	0.999					
R Square	0.998					
Adjusted R Square	0.873					
Standard Error	0.001					
	df	SS	MS	<b>F</b>	Significance F	
Regression	3	0.007	0.002	1498	3E-10	
Residual	8	1E-05	2E-06			
Total	11	0.007				
	<b>Coefficients</b>	Standard Error	<b>t Stat</b>	P-value	Lower 95%	Upper 95%
<b>b<sub>2</sub>a<sub>3</sub>E<sub>32</sub></b>	-0.0076	0.001	-12.9	1E-06	-0.009	-0.006
<b>b<sub>3</sub></b>	0.0114	0.001	18.6	7E-08	0.010	0.013
<b>a<sub>3</sub>E<sub>31</sub></b>	0.9325	0.070	13.4	9E-07	0.772	1.093

Table 13. Output of the fittest condition (B)

Calculated  $E_{ij}$  are :

$$\begin{aligned}
 Y_1 : \quad & b_1 = 0.0230 \quad b_2 a_1 E_{12} = 0.0234 \quad \Rightarrow \quad E_{12} = 2.5531 \\
 & a_1 = 0.9770, \quad b_3 a_1 E_{13} = -0.0014 \quad \Rightarrow \quad E_{13} = -0.1274 \\
 \\ 
 Y_2 : \quad & b_2 = 0.0094, \quad a_2 E_{21} = -0.0378 \quad \Rightarrow \quad E_{21} = -0.0381 \\
 & a_2 = 0.9906, \quad b_3 a_2 E_{23} = 0.0001 \quad \Rightarrow \quad E_{23} = 0.0108 \\
 \\ 
 Y_3 : \quad & b_3 = 0.0114, \quad a_3 E_{31} = -0.9325 \quad \Rightarrow \quad E_{31} = -0.9432 \\
 & a_3 = 0.9886, \quad b_2 a_3 E_{32} = 0.0076 \quad \Rightarrow \quad E_{32} = 0.8211
 \end{aligned}$$

	U.S.	Japan	U.S.S.R
U.S.	—	(2.5531)	<b>-0.1274</b>
Japan	(-0.0381)	—	(0.0108)
U.S.S.R	<b>-0.9432</b>	(0.8211)	—

Table 14.  $E_{ij}$  in Regression (B2)

Decision rules for two statistical tests are the same as those in the Nash Equilibrium case<sup>20</sup>.

- F-Test : If  $F > 4.07$ , a regression relation exists.
- $t$ -Test : If  $|t| > 2.306$ , a coefficient is fit.

<sup>20</sup> Because degrees of Freedom between the two simulations are the same, F and  $t$  distributions for decision rules are the same as well.

With these rules, it may be concluded that the trend in the results on a Leader - Follower situation is the same as that on the Nash equilibrium in general. The trend is again as follows :

- Except for two coefficients  $a_2E_{21}$  and  $b_3a_2E_{23}$  , there is a regression relationship in each coefficient, and sets of them, statistically,
- $E_{13}$  and  $E_{31}$  are fit to the Commitment Based Model of Defense Allies and Adversaries when  $Y_1$  and  $Y_3$  are reduced to fifteen percent of their full amounts,
- The other  $E_{ij}$  without  $E_{13}$  and  $E_{31}$  may be affected by  $Y_2$ .

Although one coefficient for (9.B) deteriorates in comparison with those of the Nash equilibrium, all coefficients for (9.C) and two coefficients for (9.B) improve to the contrary. This means that the regression models for followers (9.B) and (9.C) are fitter than the regression models for the Nash case.

Thus, the results of the analysis of the simulation in this section are as follows :

- a. Japanese defense expenditures do not conform to the Commitment-Based Model of Defense Allies and Adversaries on a Leader - Follower situation.
- b. However, a Leader - Follower situation better fits to the three countries in the Commitment Based Model of Defense Allies and Adversaries than the Nash case.

## V. CONCLUSIONS

The analysis of the simulation using spreadsheets may be summarized as follows :

- Reduction in adversarial intensity between the U.S. and the Soviet Union/ Russia occurred post-1991( $E_{13}=E_{31}=E_{32}$  negative value decreased.). This reduction was related to the collapse of the Soviet Union in 1991.
- Reduction is evident in how the Japanese perceived the U.S. commitments and the Soviet / Russian threats post 1987 ( $E_{21}$  and  $E_{23}$  went down.). These reductions resulted from a decrease in the Soviet defense expenditures (absolute value) after 1988 and the inversion of superiority between the Soviet Union and the U.S. in the amount of defense expenditures after 1989.
- Reemergence of Japanese perceived threats from Russia occurred post-1991( $E_{23}$  negative value rose.). This was caused by a reduction in adversarial intensity between the U.S. and the Soviet Union/Russia.

During 1983~1993 the levels of commitments and threats among the three countries fluctuated according to changes in the international security environment and defense expenditures in the U.S. and the Soviet Union/Russia. However, an odd fact of Japanese defense expenditures in Figures 2 and 3 should be noted. It is that Japanese defense expenditures, both in an absolute amount and as a percent of GNP, were not affected by these dramatic changes, while the U.S. and the Soviet/Russian defense expenditures were clearly affected. As shown in Chapter II, the amount of Japanese defense expenditures increased at a rate of 3.8 percent constantly throughout these years and the ratio of Japanese defense expenditures as a percent of GNP was constant at 1.0 percent.

Similar results are derived from Nash-based regression model as well as Leader - Follower-based regression model. Observations from the analyses are as follows :

- Japanese defense expenditures cannot be explained by the Commitment Based Model of Defense Allies and Adversaries using Nash or Leader - Follower frameworks.

- Japanese defense expenditures in 1983 ~ 1993 may have been based on reasons other than selective security considerations related to the U.S. and the Soviet Union/Russia, while the U.S. and the Soviet Union/Russia clearly impacted each other.

The Commitment-Based Model of Defense Allies and Adversaries reveals that Japanese defense policy viewed from the perspective of defense expenditures is difficult to explain in terms of relative power-politics during the transition period of this era.

## APPENDIX A. CALCULATION OF A COMMITMENT-BASED MODEL OF DEFENSE ALLIES AND ADVERSARIES

The equation (2) have been employed as a utility function with its corresponding resource constraint.

$$U_i = X_i^{A_i} Z_i^{B_i}$$

subject to  $G_i - X_i - P_i Y_i = 0$

where  $Z_i = \sum_j E_{ij} Y_j$

To maximize the utility function (2), the Lagrangean-function is used.

$$L = X_i^{A_i} Z_i^{B_i} + \lambda (G_i - X_i - P_i Y_i)$$

By differentiation with respect to  $X_i$ ,  $Y_i$  and  $\lambda$ , three equations are prepared as follows :

$$\textcircled{1} \quad L_{X_i} = Z_i^{B_i} A_i (X_i)^{A_i-1} - \lambda = 0$$

$$\textcircled{2} \quad L_{Y_i} = X_i^{A_i} B_i Z_i^{B_i-1} - \lambda P_i = 0$$

$$\textcircled{3} \quad L_{\lambda} = G_i - X_i - P_i Y_i = 0$$

Dividing  $\textcircled{1}$  by  $\textcircled{2}$ , results in ;

$$\begin{aligned} \frac{L_{X_i}}{L_{Y_i}} &= \frac{Z_i^{B_i} A_i X_i^{A_i-1}}{X_i^{A_i} B_i Z_i^{B_i-1}} = \frac{\lambda}{\lambda P_i} \\ &= \frac{A_i Z_i}{B_i X_i} = \frac{1}{P_i} \end{aligned}$$

$$\textcircled{4} \quad X_i = \frac{P_i A_i Z_i}{B_i}$$

③ is transformed by ④.

$$\begin{aligned} Y_i &= \frac{G_i - X_i}{P_i} = \frac{G_i}{P_i} - \frac{A_i Z_i}{B_i} = \frac{G_i}{P_i} - \frac{A_i}{B_i} \sum_j E_{ij} Y_j \\ &= \frac{G_i}{P_i} - \frac{A_i}{B_i} \left( Y_i + \sum_{i \neq j} E_{ij} Y_j \right) \end{aligned}$$

$$\begin{aligned} Y_i + \frac{A_i}{B_i} Y_i &= \frac{G_i}{P_i} - \frac{A_i}{B_i} \sum_{i \neq j} E_{ij} Y_j \\ Y_i \left( 1 + \frac{A_i}{B_i} \right) &= Y_i \left( \frac{B_i + A_i}{B_i} \right) = \frac{G_i}{P_i} - \frac{A_i}{B_i} \sum_{i \neq j} E_{ij} Y_j \\ Y_i &= \left( \frac{B_i}{A_i + B_i} \right) \frac{G_i}{P_i} - \left( \frac{A_i}{A_i + B_i} \right) \sum_{i \neq j} E_{ij} Y_j \end{aligned}$$

where  $\sum_j E_{ij} Y_j = Y_i + \sum_{i \neq j} E_{ij} Y_j$

Finally, the equation (3) is obtained as :

$$Y_i = \left[ \frac{A_i}{A_i + B_i} \right] \left[ \left( \frac{B_i}{A_i} \right) \left( \frac{G_i}{P_i} \right) - \sum_j E_{ij} Y_j \right] \quad (i \neq j)$$

## APPENDIX B. RESULTS OF SPREADSHEET ANALYSIS

1.

1983

i	A <sub>i</sub>	B <sub>i</sub>	P <sub>i</sub>	G	a <sub>i</sub>	b	Y <sub>i</sub>	GNP	DE
1 (U.S.)	0.4	0.01	1.0	10.0	0.98	0.02	<b>0.66</b>	4663000	308800
2 (Japan)	0.4	0.01	1.0	6.3	0.98	0.02	<b>0.06</b>	2932000	28630
3 (USSR)	0.4	0.0253	1.0	5.8	0.94	0.06	<b>0.76</b>	2728000	355100

E <sub>12</sub>	E <sub>13</sub>	E <sub>21</sub>	E <sub>23</sub>	E <sub>31</sub>	E <sub>32</sub>	D	Y <sub>1</sub> '	Y <sub>2</sub> '	Y <sub>3</sub> '
0.3	-0.6	0.0	-0.4	-0.6	-0.6	0.51	0.70	0.57	1.06
0.3	-0.6	0.1	-0.4	-0.6	-0.6	0.52	0.70	0.48	1.01
0.3	-0.6	0.2	-0.4	-0.6	-0.6	0.52	0.69	0.39	0.96
0.3	-0.6	0.3	-0.4	-0.6	-0.6	0.52	0.69	0.31	0.91
0.3	-0.6	0.4	-0.4	-0.6	-0.6	0.53	0.68	0.22	0.86
0.3	-0.6	0.5	-0.4	-0.6	-0.6	0.53	0.68	0.14	0.81
0.3	-0.6	0.6	-0.4	-0.6	-0.6	0.54	0.67	0.06	0.76
0.3	-0.6	0.7	-0.4	-0.6	-0.6	0.54	0.67	-0.03	0.71
0.3	-0.6	0.8	-0.4	-0.6	-0.6	0.54	0.66	-0.11	0.66
0.3	-0.6	0.9	-0.4	-0.6	-0.6	0.55	0.66	-0.18	0.62
0.3	-0.6	1.0	-0.4	-0.6	-0.6	0.55	0.65	-0.26	0.57
0.4	-0.6	0.0	-0.3	-0.6	-0.6	0.51	0.70	0.57	1.06
0.4	-0.6	0.1	-0.3	-0.6	-0.6	0.56	0.64	0.36	0.91
0.4	-0.6	0.2	-0.3	-0.6	-0.6	0.56	0.64	0.28	0.87
0.4	-0.6	0.3	-0.3	-0.6	-0.6	0.55	0.65	0.21	0.83
0.4	-0.6	0.4	-0.3	-0.6	-0.6	0.55	0.66	0.13	0.79
0.4	-0.6	0.5	-0.3	-0.6	-0.6	0.54	0.66	0.05	0.75
0.4	-0.6	0.6	-0.3	-0.6	-0.6	0.53	0.67	-0.03	0.71
0.4	-0.6	0.7	-0.3	-0.6	-0.6	0.53	0.68	-0.12	0.67
0.4	-0.6	0.8	-0.3	-0.6	-0.6	0.52	0.69	-0.20	0.62
0.4	-0.6	0.9	-0.3	-0.6	-0.6	0.52	0.69	-0.29	0.58
0.4	-0.6	1.0	-0.3	-0.6	-0.6	0.51	0.70	-0.38	0.53
0.4	-0.6	0.0	-0.4	-0.6	-0.6	0.54	0.62	0.55	1.01
0.4	-0.6	0.1	-0.4	-0.6	-0.6	0.53	0.63	0.47	0.97
0.4	-0.6	0.2	-0.4	-0.6	-0.6	0.52	0.63	0.39	0.93
0.4	-0.6	0.3	-0.4	-0.6	-0.6	0.52	0.64	0.31	0.88
0.4	-0.6	0.4	-0.4	-0.6	-0.6	0.51	0.65	0.23	0.84
0.4	-0.6	0.5	-0.4	-0.6	-0.6	0.51	0.66	0.15	0.80
<b>0.4</b>	<b>-0.6</b>	<b>0.6</b>	<b>-0.4</b>	<b>-0.6</b>	<b>-0.6</b>	<b>0.50</b>	<b>0.66</b>	<b>0.06</b>	<b>0.76</b>
0.4	-0.6	0.7	-0.4	-0.6	-0.6	0.49	0.67	-0.03	0.71
0.4	-0.6	0.8	-0.4	-0.6	-0.6	0.49	0.68	-0.12	0.66
0.4	-0.6	0.9	-0.4	-0.6	-0.6	0.48	0.69	-0.21	0.62
0.4	-0.6	1.0	-0.4	-0.6	-0.6	0.48	0.70	-0.30	0.57
0.5	-0.6	0.0	-0.3	-0.6	-0.6	0.51	0.70	0.57	1.06
0.5	-0.6	0.1	-0.3	-0.6	-0.6	0.57	0.59	0.35	0.88
0.5	-0.6	0.2	-0.3	-0.6	-0.6	0.55	0.60	0.28	0.85
0.5	-0.6	0.3	-0.3	-0.6	-0.6	0.54	0.62	0.21	0.82
0.5	-0.6	0.4	-0.3	-0.6	-0.6	0.52	0.64	0.13	0.78
0.5	-0.6	0.5	-0.3	-0.6	-0.6	0.51	0.66	0.05	0.75
0.5	-0.6	0.6	-0.3	-0.6	-0.6	0.49	0.68	-0.03	0.71
0.5	-0.6	0.7	-0.3	-0.6	-0.6	0.48	0.70	-0.13	0.67
0.5	-0.6	0.8	-0.3	-0.6	-0.6	0.46	0.72	-0.23	0.63
0.5	-0.6	0.9	-0.3	-0.6	-0.6	0.45	0.75	-0.33	0.58
0.5	-0.6	1.0	-0.3	-0.6	-0.6	0.43	0.77	-0.44	0.53

2.

1984

i	A <sub>i</sub>	B <sub>i</sub>	P <sub>i</sub>	G	a <sub>i</sub>	b	Y <sub>i</sub>	GNP	DE
1 (U.S.)	0.4	0.01	1.0	10.0	0.98	0.02	<b>0.62</b>	5153000	321500
2 (Japan)	0.4	0.01	1.0	5.9	0.98	0.02	<b>0.06</b>	3059000	29820
3 (USSR)	0.4	0.024	1.0	5.3	0.94	0.06	<b>0.69</b>	2750000	357600

E <sub>12</sub>	E <sub>13</sub>	E <sub>21</sub>	E <sub>23</sub>	E <sub>31</sub>	E <sub>32</sub>	D	Y <sub>1</sub> '	Y <sub>2</sub> '	Y <sub>3</sub> '
0.3	-0.6	0.0	-0.6	-0.6	-0.6	0.43	0.68	0.82	1.15
0.3	-0.6	0.1	-0.6	-0.6	-0.6	0.44	0.67	0.72	1.09
0.3	-0.6	0.2	-0.6	-0.6	-0.6	0.44	0.66	0.62	1.03
0.3	-0.6	0.3	-0.6	-0.6	-0.6	0.45	0.66	0.52	0.97
0.3	-0.6	0.4	-0.6	-0.6	-0.6	0.45	0.65	0.42	0.91
0.3	-0.6	0.5	-0.6	-0.6	-0.6	0.45	0.65	0.33	0.86
0.3	-0.6	0.6	-0.6	-0.6	-0.6	0.46	0.64	0.24	0.80
0.3	-0.6	0.7	-0.6	-0.6	-0.6	0.46	0.64	0.15	0.75
0.3	-0.6	0.8	-0.6	-0.6	-0.6	0.46	0.63	0.06	0.69
0.3	-0.6	0.9	-0.6	-0.6	-0.6	0.47	0.63	-0.03	0.64
0.3	-0.6	1.0	-0.6	-0.6	-0.6	0.47	0.62	-0.12	0.59
0.4	-0.6	0.0	-0.5	-0.6	-0.6	0.43	0.68	0.82	1.15
0.4	-0.6	0.1	-0.5	-0.6	-0.6	0.49	0.58	0.55	0.94
0.4	-0.6	0.2	-0.5	-0.6	-0.6	0.49	0.59	0.47	0.90
0.4	-0.6	0.3	-0.5	-0.6	-0.6	0.48	0.59	0.39	0.86
0.4	-0.6	0.4	-0.5	-0.6	-0.6	0.48	0.60	0.31	0.82
0.4	-0.6	0.5	-0.5	-0.6	-0.6	0.47	0.61	0.23	0.77
0.4	-0.6	0.6	-0.5	-0.6	-0.6	0.47	0.62	0.14	0.73
0.4	-0.6	0.7	-0.5	-0.6	-0.6	0.46	0.62	0.05	0.69
0.4	-0.6	0.8	-0.5	-0.6	-0.6	0.45	0.63	-0.04	0.64
0.4	-0.6	0.9	-0.5	-0.6	-0.6	0.45	0.64	-0.13	0.59
0.4	-0.6	1.0	-0.5	-0.6	-0.6	0.44	0.65	-0.22	0.54
0.4	-0.6	0.0	-0.6	-0.6	-0.6	0.47	0.56	0.76	1.05
0.4	-0.6	0.1	-0.6	-0.6	-0.6	0.46	0.57	0.68	1.01
0.4	-0.6	0.2	-0.6	-0.6	-0.6	0.46	0.58	0.60	0.97
0.4	-0.6	0.3	-0.6	-0.6	-0.6	0.45	0.58	0.51	0.92
0.4	-0.6	0.4	-0.6	-0.6	-0.6	0.44	0.59	0.43	0.88
0.4	-0.6	0.5	-0.6	-0.6	-0.6	0.44	0.60	0.34	0.83
0.4	-0.6	0.6	-0.6	-0.6	-0.6	0.43	0.61	0.25	0.79
0.4	-0.6	0.7	-0.6	-0.6	-0.6	0.43	0.62	0.16	0.74
<b>0.4</b>	<b>-0.6</b>	<b>0.8</b>	<b>-0.6</b>	<b>-0.6</b>	<b>-0.6</b>	0.42	<b>0.62</b>	<b>0.06</b>	<b>0.69</b>
0.4	-0.6	0.9	-0.6	-0.6	-0.6	0.41	0.63	-0.04	0.64
0.4	-0.6	1.0	-0.6	-0.6	-0.6	0.41	0.64	-0.14	0.59
0.5	-0.6	0.0	-0.5	-0.6	-0.6	0.43	0.68	0.82	1.15
0.5	-0.6	0.1	-0.5	-0.6	-0.6	0.51	0.51	0.53	0.89
0.5	-0.6	0.2	-0.5	-0.6	-0.6	0.50	0.52	0.46	0.86
0.5	-0.6	0.3	-0.5	-0.6	-0.6	0.48	0.54	0.39	0.83
0.5	-0.6	0.4	-0.5	-0.6	-0.6	0.47	0.56	0.32	0.80
0.5	-0.6	0.5	-0.5	-0.6	-0.6	0.45	0.57	0.24	0.76
0.5	-0.6	0.6	-0.5	-0.6	-0.6	0.44	0.59	0.15	0.72
0.5	-0.6	0.7	-0.5	-0.6	-0.6	0.42	0.62	0.06	0.68
0.5	-0.6	0.8	-0.5	-0.6	-0.6	0.41	0.64	-0.04	0.64
0.5	-0.6	0.9	-0.5	-0.6	-0.6	0.39	0.66	-0.15	0.59
0.5	-0.6	1.0	-0.5	-0.6	-0.6	0.37	0.69	-0.26	0.54

3.

1985

i	A <sub>i</sub>	B <sub>i</sub>	P <sub>i</sub>	G <sub>i</sub>	a <sub>i</sub>	b <sub>i</sub>	Y <sub>i</sub>	GNP	DE
1 (U.S.)	0.4	0.0104	1.0	10.0	0.97	0.03	<b>0.64</b>	5304000	337800
2 (Japan)	0.4	0.01	1.0	6.1	0.98	0.02	<b>0.06</b>	3217000	31390
3 (USSR)	0.4	0.0235	1.0	5.2	0.94	0.06	<b>0.68</b>	2771000	362700

E <sub>12</sub>	E <sub>13</sub>	E <sub>21</sub>	E <sub>23</sub>	E <sub>31</sub>	E <sub>32</sub>	D	Y <sub>1</sub>	Y <sub>2</sub>	Y <sub>3</sub>
0.2	-0.6	0.0	-0.6	-0.6	-0.6	0.40	0.81	0.88	1.25
0.2	-0.6	0.1	-0.6	-0.6	-0.6	0.41	0.79	0.75	1.16
0.2	-0.6	0.2	-0.6	-0.6	-0.6	0.43	0.76	0.63	1.08
0.2	-0.6	0.3	-0.6	-0.6	-0.6	0.44	0.74	0.52	1.00
0.2	-0.6	0.4	-0.6	-0.6	-0.6	0.45	0.72	0.41	0.93
0.2	-0.6	0.5	-0.6	-0.6	-0.6	0.47	0.70	0.31	0.86
0.2	-0.6	0.6	-0.6	-0.6	-0.6	0.48	0.68	0.22	0.80
0.2	-0.6	0.7	-0.6	-0.6	-0.6	0.49	0.66	0.13	0.74
0.2	-0.6	0.8	-0.6	-0.6	-0.6	0.51	0.64	0.04	0.68
0.2	-0.6	0.9	-0.6	-0.6	-0.6	0.52	0.63	-0.04	0.62
0.2	-0.6	1.0	-0.6	-0.6	-0.6	0.53	0.61	-0.11	0.57
0.3	-0.6	0.0	-0.6	-0.6	-0.6	0.40	0.81	0.88	1.25
0.3	-0.6	0.1	-0.6	-0.6	-0.6	0.44	0.67	0.71	1.08
0.3	-0.6	0.2	-0.6	-0.6	-0.6	0.44	0.67	0.61	1.02
0.3	-0.6	0.3	-0.6	-0.6	-0.6	0.45	0.66	0.51	0.96
0.3	-0.6	0.4	-0.6	-0.6	-0.6	0.45	0.66	0.42	0.90
0.3	-0.6	0.5	-0.6	-0.6	-0.6	0.45	0.65	0.32	0.84
0.3	-0.6	0.6	-0.6	-0.6	-0.6	0.46	0.65	0.23	0.79
0.3	-0.6	0.7	-0.6	-0.6	-0.6	0.46	0.64	0.14	0.73
0.3	-0.6	0.8	-0.6	-0.6	-0.6	0.46	0.64	0.05	0.68
0.3	-0.6	0.9	-0.6	-0.6	-0.6	0.47	0.63	-0.04	0.62
0.3	-0.6	1.0	-0.6	-0.6	-0.6	0.47	0.63	-0.13	0.57
0.3	-0.6	0.0	-0.7	-0.6	-0.6	0.39	0.69	1.00	1.25
0.3	-0.6	0.1	-0.7	-0.6	-0.6	0.40	0.68	0.89	1.18
0.3	-0.6	0.2	-0.7	-0.6	-0.6	0.40	0.68	0.78	1.11
0.3	-0.6	0.3	-0.7	-0.6	-0.6	0.41	0.67	0.67	1.05
0.3	-0.6	0.4	-0.7	-0.6	-0.6	0.41	0.67	0.56	0.98
0.3	-0.6	0.5	-0.7	-0.6	-0.6	0.41	0.66	0.46	0.92
0.3	-0.6	0.6	-0.7	-0.6	-0.6	0.42	0.65	0.35	0.86
0.3	-0.6	0.7	-0.7	-0.6	-0.6	0.42	0.65	0.25	0.80
0.3	-0.6	0.8	-0.7	-0.6	-0.6	0.43	0.64	0.15	0.74
<b>0.3</b>	<b>-0.6</b>	<b>0.9</b>	<b>-0.7</b>	<b>-0.6</b>	<b>-0.6</b>	<b>0.43</b>	<b>0.64</b>	<b>0.06</b>	<b>0.68</b>
0.3	-0.6	1.0	-0.7	-0.6	-0.6	0.43	0.63	-0.04	0.62
0.4	-0.6	0.0	-0.7	-0.6	-0.6	0.40	0.81	0.88	1.25
0.4	-0.6	0.1	-0.7	-0.6	-0.6	0.43	0.56	0.83	1.08
0.4	-0.6	0.2	-0.7	-0.6	-0.6	0.42	0.57	0.74	1.03
0.4	-0.6	0.3	-0.7	-0.6	-0.6	0.42	0.58	0.65	0.99
0.4	-0.6	0.4	-0.7	-0.6	-0.6	0.41	0.58	0.56	0.94
0.4	-0.6	0.5	-0.7	-0.6	-0.6	0.40	0.59	0.47	0.89
0.4	-0.6	0.6	-0.7	-0.6	-0.6	0.40	0.60	0.37	0.84
0.4	-0.6	0.7	-0.7	-0.6	-0.6	0.39	0.61	0.27	0.79
0.4	-0.6	0.8	-0.7	-0.6	-0.6	0.39	0.62	0.17	0.74
0.4	-0.6	0.9	-0.7	-0.6	-0.6	0.38	0.63	0.06	0.68
0.4	-0.6	1.0	-0.7	-0.6	-0.6	0.38	0.64	-0.05	0.62

4.

1986

i	A <sub>i</sub>	B <sub>i</sub>	P <sub>i</sub>	G <sub>i</sub>	a <sub>i</sub>	b <sub>i</sub>	Y <sub>i</sub>	GNP	DE
1 (U.S.)	0.4	0.0114	1.0	10.0	0.97	0.03	<b>0.66</b>	5450000	357900
2 (Japan)	0.4	0.01	1.0	6.1	0.98	0.02	<b>0.06</b>	3303000	32880
3 (USSR)	0.4	0.0225	1.0	5.3	0.95	0.05	<b>0.67</b>	2867000	366400

E <sub>12</sub>	E <sub>13</sub>	E <sub>21</sub>	E <sub>23</sub>	E <sub>31</sub>	E <sub>32</sub>	D	Y <sub>1</sub> '	Y <sub>2</sub> '	Y <sub>3</sub> '
0.2	-0.6	0.0	-0.7	-0.6	-0.6	0.36	0.89	1.12	1.42
0.2	-0.6	0.1	-0.7	-0.6	-0.6	0.37	0.86	0.96	1.31
0.2	-0.6	0.2	-0.7	-0.6	-0.6	0.38	0.83	0.81	1.21
0.2	-0.6	0.3	-0.7	-0.6	-0.6	0.40	0.80	0.68	1.12
0.2	-0.6	0.4	-0.7	-0.6	-0.6	0.41	0.77	0.55	1.03
0.2	-0.6	0.5	-0.7	-0.6	-0.6	0.42	0.75	0.43	0.95
0.2	-0.6	0.6	-0.7	-0.6	-0.6	0.44	0.72	0.32	0.87
0.2	-0.6	0.7	-0.7	-0.6	-0.6	0.45	0.70	0.22	0.80
0.2	-0.6	0.8	-0.7	-0.6	-0.6	0.46	0.68	0.12	0.73
0.2	-0.6	0.9	-0.7	-0.6	-0.6	0.48	0.66	0.02	0.67
0.2	-0.6	1.0	-0.7	-0.6	-0.6	0.49	0.65	-0.07	0.61
0.3	-0.6	0.0	-0.6	-0.6	-0.6	0.36	0.89	1.12	1.42
0.3	-0.6	0.1	-0.6	-0.6	-0.6	0.44	0.70	0.71	1.08
0.3	-0.6	0.2	-0.6	-0.6	-0.6	0.44	0.70	0.61	1.02
0.3	-0.6	0.3	-0.6	-0.6	-0.6	0.44	0.69	0.51	0.96
0.3	-0.6	0.4	-0.6	-0.6	-0.6	0.45	0.68	0.41	0.90
0.3	-0.6	0.5	-0.6	-0.6	-0.6	0.45	0.68	0.31	0.84
0.3	-0.6	0.6	-0.6	-0.6	-0.6	0.46	0.67	0.21	0.78
0.3	-0.6	0.7	-0.6	-0.6	-0.6	0.46	0.67	0.12	0.73
0.3	-0.6	0.8	-0.6	-0.6	-0.6	0.46	0.66	0.02	0.67
0.3	-0.6	0.9	-0.6	-0.6	-0.6	0.47	0.65	-0.07	0.61
0.3	-0.6	1.0	-0.6	-0.6	-0.6	0.47	0.65	-0.16	0.56
0.3	-0.6	0.0	-0.7	-0.6	-0.6	0.39	0.72	1.01	1.26
0.3	-0.6	0.1	-0.7	-0.6	-0.6	0.40	0.71	0.89	1.19
0.3	-0.6	0.2	-0.7	-0.6	-0.6	0.40	0.70	0.78	1.12
0.3	-0.6	0.3	-0.7	-0.6	-0.6	0.41	0.70	0.66	1.05
0.3	-0.6	0.4	-0.7	-0.6	-0.6	0.41	0.69	0.55	0.99
0.3	-0.6	0.5	-0.7	-0.6	-0.6	0.41	0.69	0.44	0.92
0.3	-0.6	0.6	-0.7	-0.6	-0.6	0.42	0.68	0.34	0.86
0.3	-0.6	0.7	-0.7	-0.6	-0.6	0.42	0.67	0.23	0.79
0.3	-0.6	0.8	-0.7	-0.6	-0.6	0.42	0.67	0.13	0.73
<b>0.3</b>	<b>-0.6</b>	<b>0.9</b>	<b>-0.7</b>	<b>-0.6</b>	<b>-0.6</b>	<b>0.43</b>	<b>0.66</b>	<b>0.03</b>	<b>0.67</b>
0.3	-0.6	1.0	-0.7	-0.6	-0.6	0.43	0.65	-0.07	0.61
0.3	-0.6	0.0	-0.8	-0.6	-0.6	0.36	0.89	1.12	1.42
0.3	-0.6	0.1	-0.8	-0.6	-0.6	0.36	0.72	1.11	1.32
0.3	-0.6	0.2	-0.8	-0.6	-0.6	0.36	0.72	0.98	1.24
0.3	-0.6	0.3	-0.8	-0.6	-0.6	0.37	0.71	0.85	1.17
0.3	-0.6	0.4	-0.8	-0.6	-0.6	0.37	0.70	0.73	1.09
0.3	-0.6	0.5	-0.8	-0.6	-0.6	0.37	0.69	0.60	1.02
0.3	-0.6	0.6	-0.8	-0.6	-0.6	0.38	0.69	0.48	0.95
0.3	-0.6	0.7	-0.8	-0.6	-0.6	0.38	0.68	0.37	0.87
0.3	-0.6	0.8	-0.8	-0.6	-0.6	0.39	0.67	0.25	0.81
0.3	-0.6	0.9	-0.8	-0.6	-0.6	0.39	0.67	0.14	0.74
0.3	-0.6	1.0	-0.8	-0.6	-0.6	0.39	0.66	0.03	0.67

5.

1987

i	A <sub>i</sub>	B <sub>i</sub>	P <sub>i</sub>	G <sub>i</sub>	a <sub>i</sub>	b <sub>i</sub>	Y <sub>i</sub>	GNP	DE
1 (U.S.)	0.4	0.0103	1.0	10.0	0.97	0.03	<b>0.63</b>	5612000	355800
2 (Japan)	0.4	0.01	1.0	6.1	0.98	0.02	<b>0.06</b>	3446000	34580
3 (USSR)	0.4	0.0233	1.0	5.2	0.94	0.06	<b>0.67</b>	2899000	374200

E <sub>12</sub>	E <sub>13</sub>	E <sub>21</sub>	E <sub>23</sub>	E <sub>31</sub>	E <sub>32</sub>	D	Y <sub>1</sub> '	Y <sub>2</sub> '	Y <sub>3</sub> '
0.3	-0.6	0.0	-0.6	-0.6	-0.6	0.43	0.67	0.81	1.12
0.3	-0.6	0.1	-0.6	-0.6	-0.6	0.44	0.67	0.71	1.06
0.3	-0.6	0.2	-0.6	-0.6	-0.6	0.44	0.66	0.61	1.00
0.3	-0.6	0.3	-0.6	-0.6	-0.6	0.45	0.65	0.51	0.95
0.3	-0.6	0.4	-0.6	-0.6	-0.6	0.45	0.65	0.42	0.89
0.3	-0.6	0.5	-0.6	-0.6	-0.6	0.45	0.64	0.32	0.83
0.3	-0.6	0.6	-0.6	-0.6	-0.6	0.46	0.64	0.23	0.78
0.3	-0.6	0.7	-0.6	-0.6	-0.6	0.46	0.63	0.14	0.72
0.3	-0.6	0.8	-0.6	-0.6	-0.6	0.46	0.63	0.05	0.67
0.3	-0.6	0.9	-0.6	-0.6	-0.6	0.47	0.62	-0.04	0.62
0.3	-0.6	1.0	-0.6	-0.6	-0.6	0.47	0.62	-0.12	0.57
0.3	-0.6	0.0	-0.7	-0.6	-0.6	0.43	0.67	0.81	1.12
0.3	-0.6	0.1	-0.7	-0.6	-0.6	0.40	0.68	0.88	1.17
0.3	-0.6	0.2	-0.7	-0.6	-0.6	0.40	0.67	0.77	1.10
0.3	-0.6	0.3	-0.7	-0.6	-0.6	0.41	0.66	0.66	1.04
0.3	-0.6	0.4	-0.7	-0.6	-0.6	0.41	0.66	0.56	0.97
0.3	-0.6	0.5	-0.7	-0.6	-0.6	0.41	0.65	0.45	0.91
0.3	-0.6	0.6	-0.7	-0.6	-0.6	0.42	0.65	0.35	0.85
0.3	-0.6	0.7	-0.7	-0.6	-0.6	0.42	0.64	0.25	0.79
0.3	-0.6	0.8	-0.7	-0.6	-0.6	0.42	0.63	0.15	0.73
<b>0.3</b>	<b>-0.6</b>	<b>0.9</b>	<b>-0.7</b>	<b>-0.6</b>	<b>-0.6</b>	0.43	<b>0.63</b>	<b>0.06</b>	<b>0.67</b>
0.3	-0.6	1.0	-0.7	-0.6	-0.6	0.43	0.62	-0.04	0.62
0.4	-0.6	0.0	-0.6	-0.6	-0.6	0.47	0.56	0.75	1.03
0.4	-0.6	0.1	-0.6	-0.6	-0.6	0.46	0.57	0.67	0.99
0.4	-0.6	0.2	-0.6	-0.6	-0.6	0.45	0.57	0.59	0.94
0.4	-0.6	0.3	-0.6	-0.6	-0.6	0.45	<b>0.58</b>	0.51	0.90
0.4	-0.6	0.4	-0.6	-0.6	-0.6	0.44	0.59	0.42	0.86
0.4	-0.6	0.5	-0.6	-0.6	-0.6	0.44	0.60	0.33	0.81
0.4	-0.6	0.6	-0.6	-0.6	-0.6	0.43	0.60	0.24	0.76
0.4	-0.6	0.7	-0.6	-0.6	-0.6	0.43	0.61	0.15	0.72
0.4	-0.6	0.8	-0.6	-0.6	-0.6	0.42	0.62	0.06	0.67
0.4	-0.6	0.9	-0.6	-0.6	-0.6	0.41	0.63	-0.04	0.62
0.4	-0.6	1.0	-0.6	-0.6	-0.6	0.41	0.64	-0.14	0.57
0.4	-0.6	0.0	-0.7	-0.6	-0.6	0.43	0.67	0.81	1.12
0.4	-0.6	0.1	-0.7	-0.6	-0.6	0.43	0.55	0.82	1.06
0.4	-0.6	0.2	-0.7	-0.6	-0.6	0.42	0.56	0.74	1.02
0.4	-0.6	0.3	-0.7	-0.6	-0.6	0.42	0.57	0.65	0.97
0.4	-0.6	0.4	-0.7	-0.6	-0.6	0.41	0.58	0.56	0.93
0.4	-0.6	0.5	-0.7	-0.6	-0.6	0.40	<b>0.58</b>	0.47	0.88
0.4	-0.6	0.6	-0.7	-0.6	-0.6	0.40	0.59	0.37	0.83
0.4	-0.6	0.7	-0.7	-0.6	-0.6	0.39	0.60	0.27	0.78
0.4	-0.6	0.8	-0.7	-0.6	-0.6	0.39	0.61	0.17	0.73
0.4	-0.6	0.9	-0.7	-0.6	-0.6	0.38	0.62	0.07	0.67
0.4	-0.6	1.0	-0.7	-0.6	-0.6	<b>0.38</b>	0.63	-0.04	0.62

6.

1988

i	A <sub>i</sub>	B <sub>i</sub>	P <sub>i</sub>	Q <sub>i</sub>	a <sub>i</sub>	b <sub>i</sub>	Y <sub>i</sub>	GNP	DE
1 (U.S.)	0.4	0.01	1.0	10.0	0.98	0.02	<b>0.60</b>	3837000	348500
2 (Japan)	0.4	0.01	1.0	6.3	0.98	0.02	<b>0.06</b>	3661000	36250
3 (USSR)	0.4	0.0222	1.0	5.1	0.95	0.05	<b>0.65</b>	2982000	379300

E <sub>12</sub>	E <sub>13</sub>	E <sub>21</sub>	E <sub>23</sub>	E <sub>31</sub>	E <sub>32</sub>	D	Y <sub>1</sub>	Y <sub>2</sub>	Y <sub>3</sub>
0.2	-0.6	0.0	-0.6	-0.6	-0.6	0.40	0.78	0.85	1.19
0.2	-0.6	0.1	-0.6	-0.6	-0.6	0.41	0.75	0.73	1.11
0.2	-0.6	0.2	-0.6	-0.6	-0.6	0.43	0.73	0.62	1.03
0.2	-0.6	0.3	-0.6	-0.6	-0.6	0.44	0.71	0.51	0.96
0.2	-0.6	0.4	-0.6	-0.6	-0.6	0.45	0.68	0.41	0.89
0.2	-0.6	0.5	-0.6	-0.6	-0.6	0.47	0.67	0.31	0.82
0.2	-0.6	0.6	-0.6	-0.6	-0.6	0.48	0.65	0.22	0.76
0.2	-0.6	0.7	-0.6	-0.6	-0.6	0.49	0.63	0.13	0.70
0.2	-0.6	0.8	-0.6	-0.6	-0.6	0.51	0.61	0.05	0.65
0.2	-0.6	0.9	-0.6	-0.6	-0.6	0.52	0.60	-0.02	0.59
0.2	-0.6	1.0	-0.6	-0.6	-0.6	0.53	0.58	-0.10	0.55
0.3	-0.6	0.0	-0.5	-0.6	-0.6	0.40	0.78	0.85	1.19
0.3	-0.6	0.1	-0.5	-0.6	-0.6	0.48	0.63	0.55	0.94
0.3	-0.6	0.2	-0.5	-0.6	-0.6	0.48	0.63	0.46	0.89
0.3	-0.6	0.3	-0.5	-0.6	-0.6	0.48	0.62	0.38	0.84
0.3	-0.6	0.4	-0.5	-0.6	-0.6	0.49	0.62	0.30	0.79
0.3	-0.6	0.5	-0.5	-0.6	-0.6	0.49	0.61	0.21	0.74
0.3	-0.6	0.6	-0.5	-0.6	-0.6	0.49	0.61	0.13	0.69
0.3	-0.6	0.7	-0.5	-0.6	-0.6	0.50	0.60	0.05	0.64
0.3	-0.6	0.8	-0.5	-0.6	-0.6	0.50	0.60	-0.02	0.60
0.3	-0.6	0.9	-0.5	-0.6	-0.6	0.51	0.60	-0.10	0.55
0.3	-0.6	1.0	-0.5	-0.6	-0.6	0.51	0.59	-0.18	0.50
0.3	-0.6	0.0	-0.6	-0.6	-0.6	0.43	0.65	0.79	1.09
0.3	-0.6	0.1	-0.6	-0.6	-0.6	0.44	0.64	0.69	1.03
0.3	-0.6	0.2	-0.6	-0.6	-0.6	0.44	0.64	0.60	0.97
0.3	-0.6	0.3	-0.6	-0.6	-0.6	0.44	0.63	0.50	0.91
0.3	-0.6	0.4	-0.6	-0.6	-0.6	0.45	0.63	0.41	0.86
0.3	-0.6	0.5	-0.6	-0.6	-0.6	0.45	0.62	0.32	0.80
0.3	-0.6	0.6	-0.6	-0.6	-0.6	0.46	0.62	0.23	0.75
0.3	-0.6	0.7	-0.6	-0.6	-0.6	0.46	0.61	0.14	0.70
<b>0.3</b>	<b>-0.6</b>	<b>0.8</b>	<b>-0.6</b>	<b>-0.6</b>	<b>-0.6</b>	0.46	<b>0.60</b>	<b>0.06</b>	<b>0.65</b>
0.3	-0.6	0.9	-0.6	-0.6	-0.6	0.47	0.60	-0.03	0.60
0.3	-0.6	1.0	-0.6	-0.6	-0.6	0.47	0.59	-0.11	0.55
0.4	-0.6	0.0	-0.6	-0.6	-0.6	0.40	0.78	0.85	1.19
0.4	-0.6	0.1	-0.6	-0.6	-0.6	0.46	0.54	0.66	0.95
0.4	-0.6	0.2	-0.6	-0.6	-0.6	0.45	0.55	0.58	0.91
0.4	-0.6	0.3	-0.6	-0.6	-0.6	0.45	0.56	0.50	0.87
0.4	-0.6	0.4	-0.6	-0.6	-0.6	0.44	0.57	0.42	0.83
0.4	-0.6	0.5	-0.6	-0.6	-0.6	0.44	0.57	0.33	0.78
0.4	-0.6	0.6	-0.6	-0.6	-0.6	0.43	0.58	0.25	0.74
0.4	-0.6	0.7	-0.6	-0.6	-0.6	0.43	0.59	0.16	0.69
0.4	-0.6	0.8	-0.6	-0.6	-0.6	0.42	0.60	0.07	0.64
0.4	-0.6	0.9	-0.6	-0.6	-0.6	0.41	0.60	-0.03	0.60
0.4	-0.6	1.0	-0.6	-0.6	-0.6	0.41	0.61	-0.12	0.55

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1989

i	A <sub>i</sub>	B <sub>i</sub>	P <sub>i</sub>	G	a <sub>i</sub>	b	Y <sub>i</sub>	GNP	DE
1 (U.S.)	0.4	0.0104	1.0	10.0	0.97	0.03	<b>0.58</b>	5993000	346000
2 (Japan)	0.4	0.01	1.0	6.4	0.98	0.02	<b>0.06</b>	3838000	37700
3 (USSR)	0.4	0.0185	1.0	5.0	0.96	0.04	<b>0.58</b>	3010000	344800

E <sub>12</sub>	E <sub>13</sub>	E <sub>21</sub>	E <sub>23</sub>	E <sub>31</sub>	E <sub>32</sub>	D	Y <sub>1</sub> <sup>*</sup>	Y <sub>2</sub> <sup>*</sup>	Y <sub>3</sub> <sup>*</sup>
0.2	-0.6	0.0	-0.1	-0.6	-0.6	0.62	0.62	0.23	0.71
0.2	-0.6	0.1	-0.1	-0.6	-0.6	0.63	0.61	0.16	0.66
0.2	-0.6	0.2	-0.1	-0.6	-0.6	0.65	0.60	0.10	0.62
0.2	-0.6	0.3	-0.1	-0.6	-0.6	0.66	0.59	0.04	0.58
0.2	-0.6	0.4	-0.1	-0.6	-0.6	0.67	0.57	-0.01	0.54
0.2	-0.6	0.5	-0.1	-0.6	-0.6	0.69	0.56	-0.07	0.50
0.2	-0.6	0.6	-0.1	-0.6	-0.6	0.70	0.55	-0.12	0.47
0.2	-0.6	0.7	-0.1	-0.6	-0.6	0.72	0.54	-0.17	0.43
0.2	-0.6	0.8	-0.1	-0.6	-0.6	0.73	0.53	-0.22	0.40
0.2	-0.6	0.9	-0.1	-0.6	-0.6	0.74	0.52	-0.26	0.37
0.2	-0.6	1.0	-0.1	-0.6	-0.6	0.76	0.51	-0.31	0.34
0.3	-0.6	0.0	-0.1	-0.6	-0.6	0.62	0.62	0.23	0.71
0.3	-0.6	0.1	-0.1	-0.6	-0.6	0.63	0.59	0.16	0.65
0.3	-0.6	0.2	-0.1	-0.6	-0.6	0.63	0.58	0.10	0.62
0.3	-0.6	0.3	-0.1	-0.6	-0.6	0.64	0.58	0.04	0.58
0.3	-0.6	0.4	-0.1	-0.6	-0.6	0.64	0.58	-0.02	0.54
0.3	-0.6	0.5	-0.1	-0.6	-0.6	0.65	0.57	-0.07	0.51
0.3	-0.6	0.6	-0.1	-0.6	-0.6	0.65	0.57	-0.13	0.47
0.3	-0.6	0.7	-0.1	-0.6	-0.6	0.65	0.56	-0.19	0.44
0.3	-0.6	0.8	-0.1	-0.6	-0.6	0.66	0.56	-0.24	0.40
0.3	-0.6	0.9	-0.1	-0.6	-0.6	0.66	0.56	-0.30	0.37
0.3	-0.6	1.0	-0.1	-0.6	-0.6	0.67	0.55	-0.35	0.34
0.3	-0.6	0.0	-0.2	-0.6	-0.6	0.59	0.60	0.30	0.74
0.3	-0.6	0.1	-0.2	-0.6	-0.6	0.59	0.59	0.23	0.70
0.3	-0.6	0.2	-0.2	-0.6	-0.6	0.59	0.59	0.17	0.66
0.3	-0.6	0.3	-0.2	-0.6	-0.6	0.60	0.58	0.11	0.62
<b>0.3</b>	<b>-0.6</b>	<b>0.4</b>	<b>-0.2</b>	<b>-0.6</b>	<b>-0.6</b>	0.60	<b>0.58</b>	<b>0.04</b>	<b>0.58</b>
0.3	-0.6	0.5	-0.2	-0.6	-0.6	0.61	0.58	-0.02	0.54
0.3	-0.6	0.6	-0.2	-0.6	-0.6	0.61	0.57	-0.08	0.50
0.3	-0.6	0.7	-0.2	-0.6	-0.6	0.61	0.57	-0.14	0.47
0.3	-0.6	0.8	-0.2	-0.6	-0.6	0.62	0.56	-0.20	0.43
0.3	-0.6	0.9	-0.2	-0.6	-0.6	0.62	0.56	-0.26	0.39
0.3	-0.6	1.0	-0.2	-0.6	-0.6	0.63	0.56	-0.32	0.36
0.4	-0.6	0.0	-0.2	-0.6	-0.6	0.62	0.62	0.23	0.71
0.4	-0.6	0.1	-0.2	-0.6	-0.6	0.59	0.56	0.23	0.68
0.4	-0.6	0.2	-0.2	-0.6	-0.6	0.59	0.56	0.17	0.64
0.4	-0.6	0.3	-0.2	-0.6	-0.6	0.58	0.57	0.11	0.61
0.4	-0.6	0.4	-0.2	-0.6	-0.6	0.58	0.57	0.05	0.58
0.4	-0.6	0.5	-0.2	-0.6	-0.6	0.57	0.58	-0.02	0.54
0.4	-0.6	0.6	-0.2	-0.6	-0.6	0.56	0.58	-0.09	0.51
0.4	-0.6	0.7	-0.2	-0.6	-0.6	0.56	0.59	-0.15	0.47
0.4	-0.6	0.8	-0.2	-0.6	-0.6	0.55	0.59	-0.22	0.44
0.4	-0.6	0.9	-0.2	-0.6	-0.6	0.55	0.60	-0.29	0.40
0.4	-0.6	1.0	-0.2	-0.6	-0.6	0.54	0.61	-0.37	0.36

8.

1990

i	A	B <sub>i</sub>	P <sub>i</sub>	G	a	b	Y <sub>i</sub>	GNP	DE
1 (U.S.)	0.4	0.0108	1.0	10.0	0.97	0.03	<b>0.55</b>	6071000	333900
2 (Japan)	0.4	0.01	1.0	6.6	0.98	0.02	<b>0.06</b>	4021000	39130
3 (USSR)	0.4	0.015	1.0	4.8	0.96	0.04	<b>0.52</b>	2901000	318400

E <sub>12</sub>	E <sub>13</sub>	E <sub>21</sub>	E <sub>23</sub>	E <sub>31</sub>	E <sub>32</sub>	D	Y <sub>1</sub> '	Y <sub>2</sub> '	Y <sub>3</sub> '
0.2	-0.6	0.0	-0.1	-0.6	-0.6	0.62	0.60	0.22	0.65
0.2	-0.6	0.1	-0.1	-0.6	-0.6	0.63	0.58	0.16	0.61
0.2	-0.6	0.2	-0.1	-0.6	-0.6	0.64	0.57	0.10	0.56
0.2	-0.6	0.3	-0.1	-0.6	-0.6	0.66	0.56	0.05	0.52
0.2	-0.6	0.4	-0.1	-0.6	-0.6	0.67	0.55	0.00	0.49
0.2	-0.6	0.5	-0.1	-0.6	-0.6	0.69	0.54	-0.06	0.45
0.2	-0.6	0.6	-0.1	-0.6	-0.6	0.70	0.53	-0.11	0.42
0.2	-0.6	0.7	-0.1	-0.6	-0.6	0.71	0.52	-0.15	0.38
0.2	-0.6	0.8	-0.1	-0.6	-0.6	0.73	0.51	-0.20	0.35
0.2	-0.6	0.9	-0.1	-0.6	-0.6	0.74	0.50	-0.24	0.32
0.2	-0.6	1.0	-0.1	-0.6	-0.6	0.76	0.49	-0.29	0.29
0.3	-0.6	0.0	-0.1	-0.6	-0.6	0.62	0.60	0.22	0.65
0.3	-0.6	0.1	-0.1	-0.6	-0.6	0.63	0.56	0.16	0.59
0.3	-0.6	0.2	-0.1	-0.6	-0.6	0.63	0.56	0.11	0.56
0.3	-0.6	0.3	-0.1	-0.6	-0.6	0.64	0.55	0.05	0.52
0.3	-0.6	0.4	-0.1	-0.6	-0.6	0.64	0.55	-0.01	0.49
0.3	-0.6	0.5	-0.1	-0.6	-0.6	0.64	0.55	-0.06	0.45
0.3	-0.6	0.6	-0.1	-0.6	-0.6	0.65	0.54	-0.11	0.42
0.3	-0.6	0.7	-0.1	-0.6	-0.6	0.65	0.54	-0.17	0.39
0.3	-0.6	0.8	-0.1	-0.6	-0.6	0.66	0.53	-0.22	0.35
0.3	-0.6	0.9	-0.1	-0.6	-0.6	0.66	0.53	-0.27	0.32
0.3	-0.6	1.0	-0.1	-0.6	-0.6	0.67	0.53	-0.32	0.29
0.3	-0.6	0.0	-0.2	-0.6	-0.6	0.58	0.57	0.29	0.67
0.3	-0.6	0.1	-0.2	-0.6	-0.6	0.59	0.57	0.23	0.63
0.3	-0.6	0.2	-0.2	-0.6	-0.6	0.59	0.56	0.17	0.59
0.3	-0.6	0.3	-0.2	-0.6	-0.6	0.60	0.56	0.11	0.56
<b>0.3</b>	<b>-0.6</b>	<b>0.4</b>	<b>-0.2</b>	<b>-0.6</b>	<b>-0.6</b>	0.60	<b>0.55</b>	<b>0.05</b>	<b>0.52</b>
0.3	-0.6	0.5	-0.2	-0.6	-0.6	0.60	0.55	-0.01	0.48
0.3	-0.6	0.6	-0.2	-0.6	-0.6	0.61	0.54	-0.07	0.45
0.3	-0.6	0.7	-0.2	-0.6	-0.6	0.61	0.54	-0.13	0.41
0.3	-0.6	0.8	-0.2	-0.6	-0.6	0.62	0.54	-0.18	0.38
0.3	-0.6	0.9	-0.2	-0.6	-0.6	0.62	0.53	-0.24	0.34
0.3	-0.6	1.0	-0.2	-0.6	-0.6	0.63	0.53	-0.29	0.31
0.4	-0.6	0.0	-0.2	-0.6	-0.6	0.62	0.60	0.22	0.65
0.4	-0.6	0.1	-0.2	-0.6	-0.6	0.59	0.53	0.23	0.61
0.4	-0.6	0.2	-0.2	-0.6	-0.6	0.58	0.54	0.17	0.58
0.4	-0.6	0.3	-0.2	-0.6	-0.6	0.58	0.54	0.11	0.55
0.4	-0.6	0.4	-0.2	-0.6	-0.6	0.57	0.55	0.05	0.52
0.4	-0.6	0.5	-0.2	-0.6	-0.6	0.57	0.55	-0.01	0.48
0.4	-0.6	0.6	-0.2	-0.6	-0.6	0.56	0.56	-0.08	0.45
0.4	-0.6	0.7	-0.2	-0.6	-0.6	0.56	0.56	-0.14	0.42
0.4	-0.6	0.8	-0.2	-0.6	-0.6	0.55	0.57	-0.21	0.38
0.4	-0.6	0.9	-0.2	-0.6	-0.6	0.55	0.57	-0.27	0.35
0.4	-0.6	1.0	-0.2	-0.6	-0.6	0.54	0.58	-0.34	0.31

9.

1991

i	A <sub>i</sub>	B <sub>i</sub>	P <sub>i</sub>	G <sub>i</sub>	a <sub>i</sub>	b <sub>i</sub>	Y <sub>i</sub>	GNP	DE
1 (U.S.)	0.4	0.0102	1.0	10.0	0.98	0.02	<b>0.49</b>	6029000	294400
2 (Japan)	0.4	0.01	1.0	7.0	0.98	0.02	<b>0.07</b>	4193000	40460
3 (USSR)	0.4	0.0115	1.0	4.4	0.97	0.03	<b>0.45</b>	2659000	273100

E <sub>12</sub>	E <sub>13</sub>	E <sub>21</sub>	E <sub>23</sub>	E <sub>31</sub>	E <sub>32</sub>	D	Y <sub>1</sub> '	Y <sub>2</sub> '	Y <sub>3</sub> '
0.2	-0.6	0.0	-0.2	-0.6	-0.6	0.57	0.55	0.29	0.61
0.2	-0.6	0.1	-0.2	-0.6	-0.6	0.58	0.54	0.23	0.57
0.2	-0.6	0.2	-0.2	-0.6	-0.6	0.60	0.52	0.17	0.53
0.2	-0.6	0.3	-0.2	-0.6	-0.6	0.61	0.51	0.12	0.49
0.2	-0.6	0.4	-0.2	-0.6	-0.6	0.62	0.50	0.06	0.45
0.2	-0.6	0.5	-0.2	-0.6	-0.6	0.64	0.49	0.01	0.42
0.2	-0.6	0.6	-0.2	-0.6	-0.6	0.65	0.48	-0.04	0.38
0.2	-0.6	0.7	-0.2	-0.6	-0.6	0.67	0.47	-0.08	0.35
0.2	-0.6	0.8	-0.2	-0.6	-0.6	0.68	0.46	-0.13	0.32
0.2	-0.6	0.9	-0.2	-0.6	-0.6	0.70	0.45	-0.17	0.29
0.2	-0.6	1.0	-0.2	-0.6	-0.6	0.71	0.44	-0.21	0.26
0.3	-0.6	0.0	-0.1	-0.6	-0.6	0.57	0.55	0.29	0.61
0.3	-0.6	0.1	-0.1	-0.6	-0.6	0.62	0.50	0.17	0.51
0.3	-0.6	0.2	-0.1	-0.6	-0.6	0.63	0.50	0.12	0.48
<b>0.3</b>	<b>-0.6</b>	<b>0.3</b>	<b>-0.1</b>	<b>-0.6</b>	<b>-0.6</b>	0.63	<b>0.49</b>	<b>0.07</b>	<b>0.45</b>
0.3	-0.6	0.4	-0.1	-0.6	-0.6	0.64	0.49	0.02	0.42
0.3	-0.6	0.5	-0.1	-0.6	-0.6	0.64	0.48	-0.03	0.39
0.3	-0.6	0.6	-0.1	-0.6	-0.6	0.65	0.48	-0.08	0.36
0.3	-0.6	0.7	-0.1	-0.6	-0.6	0.65	0.48	-0.12	0.33
0.3	-0.6	0.8	-0.1	-0.6	-0.6	0.66	0.47	-0.17	0.30
0.3	-0.6	0.9	-0.1	-0.6	-0.6	0.66	0.47	-0.22	0.27
0.3	-0.6	1.0	-0.1	-0.6	-0.6	0.67	0.47	-0.26	0.24
0.3	-0.6	0.0	-0.2	-0.6	-0.6	0.58	0.51	0.28	0.58
0.3	-0.6	0.1	-0.2	-0.6	-0.6	0.58	0.50	0.23	0.55
0.3	-0.6	0.2	-0.2	-0.6	-0.6	0.59	0.50	0.17	0.52
0.3	-0.6	0.3	-0.2	-0.6	-0.6	0.59	0.50	0.12	0.48
0.3	-0.6	0.4	-0.2	-0.6	-0.6	0.60	0.49	0.07	0.45
0.3	-0.6	0.5	-0.2	-0.6	-0.6	0.60	0.49	0.01	0.42
0.3	-0.6	0.6	-0.2	-0.6	-0.6	0.61	0.48	-0.04	0.38
0.3	-0.6	0.7	-0.2	-0.6	-0.6	0.61	0.48	-0.09	0.35
0.3	-0.6	0.8	-0.2	-0.6	-0.6	0.62	0.48	-0.14	0.32
0.3	-0.6	0.9	-0.2	-0.6	-0.6	0.62	0.47	-0.19	0.29
0.3	-0.6	1.0	-0.2	-0.6	-0.6	0.63	0.47	-0.24	0.26
0.4	-0.6	0.0	-0.1	-0.6	-0.6	0.57	0.55	0.29	0.61
0.4	-0.6	0.1	-0.1	-0.6	-0.6	0.62	0.47	0.17	0.50
0.4	-0.6	0.2	-0.1	-0.6	-0.6	0.61	0.48	0.12	0.47
0.4	-0.6	0.3	-0.1	-0.6	-0.6	0.61	0.48	0.07	0.45
0.4	-0.6	0.4	-0.1	-0.6	-0.6	0.61	0.49	0.02	0.42
0.4	-0.6	0.5	-0.1	-0.6	-0.6	0.60	0.49	-0.03	0.39
0.4	-0.6	0.6	-0.1	-0.6	-0.6	0.60	0.49	-0.08	0.36
0.4	-0.6	0.7	-0.1	-0.6	-0.6	0.59	0.50	-0.14	0.33
0.4	-0.6	0.8	-0.1	-0.6	-0.6	0.59	0.50	-0.19	0.30
0.4	-0.6	0.9	-0.1	-0.6	-0.6	0.58	0.51	-0.25	0.27
0.4	-0.6	1.0	-0.1	-0.6	-0.6	0.58	0.51	-0.30	0.24

10.

1992

i	A <sub>i</sub>	B <sub>i</sub>	P <sub>i</sub>	G <sub>i</sub>	a <sub>i</sub>	b <sub>i</sub>	Y <sub>i</sub>	GNP	DE
1 (U.S.)	0.4	0.0181	1.0	10.0	0.96	0.04	<b>0.51</b>	6157000	311800
2 (Japan)	0.4	0.01	1.0	6.9	0.98	0.02	<b>0.07</b>	4250000	41330
3 (Russia)	0.4	0.005	1.0	1.4	0.99	0.01	<b>0.24</b>	870600	145400

E <sub>12</sub>	E <sub>13</sub>	E <sub>21</sub>	E <sub>23</sub>	E <sub>31</sub>	E <sub>32</sub>	D	Y <sub>1</sub>	Y <sub>2</sub>	Y <sub>3</sub>
0.2	-0.4	0.0	-0.2	-0.4	-0.4	0.79	0.51	0.23	0.31
0.2	-0.4	0.1	-0.2	-0.4	-0.4	0.78	0.51	0.17	0.29
0.2	-0.4	0.2	-0.2	-0.4	-0.4	0.78	0.51	0.12	0.27
0.2	-0.4	0.3	-0.2	-0.4	-0.4	0.77	0.51	0.07	0.25
0.2	-0.4	0.4	-0.2	-0.4	-0.4	0.77	0.52	0.01	0.23
0.2	-0.4	0.5	-0.2	-0.4	-0.4	0.77	0.52	-0.05	0.20
0.2	-0.4	0.6	-0.2	-0.4	-0.4	0.76	0.52	-0.10	0.18
0.2	-0.4	0.7	-0.2	-0.4	-0.4	0.76	0.53	-0.16	0.16
0.2	-0.4	0.8	-0.2	-0.4	-0.4	0.76	0.53	-0.22	0.14
0.2	-0.4	0.9	-0.2	-0.4	-0.4	0.75	0.53	-0.27	0.12
0.2	-0.4	1.0	-0.2	-0.4	-0.4	0.75	0.53	-0.33	0.10
0.3	-0.4	0.0	-0.4	-0.4	-0.4	0.79	0.51	0.23	0.31
0.3	-0.4	0.1	-0.4	-0.4	-0.4	0.73	0.48	0.24	0.30
0.3	-0.4	0.2	-0.4	-0.4	-0.4	0.71	0.49	0.18	0.28
0.3	-0.4	0.3	-0.4	-0.4	-0.4	0.70	0.50	0.13	0.26
<b>0.3</b>	<b>-0.4</b>	<b>0.4</b>	<b>-0.4</b>	<b>-0.4</b>	<b>-0.4</b>	0.69	<b>0.51</b>	<b>0.07</b>	<b>0.24</b>
0.3	-0.4	0.5	-0.4	-0.4	-0.4	0.67	0.52	0.00	0.22
0.3	-0.4	0.6	-0.4	-0.4	-0.4	0.66	0.53	-0.06	0.20
0.3	-0.4	0.7	-0.4	-0.4	-0.4	0.65	0.54	-0.13	0.18
0.3	-0.4	0.8	-0.4	-0.4	-0.4	0.63	0.55	-0.20	0.16
0.3	-0.4	0.9	-0.4	-0.4	-0.4	0.62	0.56	-0.27	0.13
0.3	-0.4	1.0	-0.4	-0.4	-0.4	0.61	0.57	-0.35	0.11
0.3	-0.4	0.0	-0.5	-0.4	-0.4	0.71	0.47	0.33	0.33
0.3	-0.4	0.1	-0.5	-0.4	-0.4	0.70	0.47	0.27	0.31
0.3	-0.4	0.2	-0.5	-0.4	-0.4	0.68	0.48	0.22	0.29
0.3	-0.4	0.3	-0.5	-0.4	-0.4	0.67	0.49	0.16	0.27
0.3	-0.4	0.4	-0.5	-0.4	-0.4	0.66	0.50	0.10	0.25
0.3	-0.4	0.5	-0.5	-0.4	-0.4	0.65	0.51	0.03	0.23
0.3	-0.4	0.6	-0.5	-0.4	-0.4	0.63	0.52	-0.04	0.21
0.3	-0.4	0.7	-0.5	-0.4	-0.4	0.62	0.53	-0.11	0.19
0.3	-0.4	0.8	-0.5	-0.4	-0.4	0.61	0.55	-0.18	0.16
0.3	-0.4	0.9	-0.5	-0.4	-0.4	0.59	0.56	-0.26	0.14
0.3	-0.4	1.0	-0.5	-0.4	-0.4	0.58	0.57	-0.34	0.11
0.4	-0.4	0.0	-0.4	-0.4	-0.4	0.79	0.51	0.23	0.31
0.4	-0.4	0.1	-0.4	-0.4	-0.4	0.73	0.45	0.24	0.29
0.4	-0.4	0.2	-0.4	-0.4	-0.4	0.71	0.47	0.18	0.28
0.4	-0.4	0.3	-0.4	-0.4	-0.4	0.69	0.48	0.13	0.26
0.4	-0.4	0.4	-0.4	-0.4	-0.4	0.66	0.50	0.07	0.24
0.4	-0.4	0.5	-0.4	-0.4	-0.4	0.64	0.52	0.00	0.22
0.4	-0.4	0.6	-0.4	-0.4	-0.4	0.62	0.54	-0.07	0.20
0.4	-0.4	0.7	-0.4	-0.4	-0.4	0.60	0.56	-0.14	0.18
0.4	-0.4	0.8	-0.4	-0.4	-0.4	0.57	0.58	-0.22	0.16
0.4	-0.4	0.9	-0.4	-0.4	-0.4	0.55	0.60	-0.31	0.13
0.4	-0.4	1.0	-0.4	-0.4	-0.4	0.53	0.63	-0.40	0.11

11.

1993

i	A <sub>i</sub>	B <sub>i</sub>	P <sub>i</sub>	G	a <sub>i</sub>	b	Y <sub>i</sub>	GNP	DE
1 (U.S.)	0.4	0.0181	1.0	10.0	0.96	0.04	<b>0.47</b>	6348000	297600
2 (Japan)	0.4	0.01	1.0	6.7	0.98	0.02	<b>0.07</b>	4260000	41730
3 (Russia)	0.4	0.009	1.0	1.2	0.98	0.02	<b>0.18</b>	777400	113800

E <sub>12</sub>	E <sub>13</sub>	E <sub>21</sub>	E <sub>23</sub>	E <sub>31</sub>	E <sub>32</sub>	D	Y <sub>1</sub> '	Y <sub>2</sub> '	Y <sub>3</sub> '
0.2	-0.3	0.0	-0.2	-0.3	-0.3	0.87	0.46	0.21	0.22
0.2	-0.3	0.1	-0.2	-0.3	-0.3	0.86	0.46	0.16	0.21
0.2	-0.3	0.2	-0.2	-0.3	-0.3	0.85	0.47	0.11	0.20
0.2	-0.3	0.3	-0.2	-0.3	-0.3	0.84	0.47	0.06	0.18
0.2	-0.3	0.4	-0.2	-0.3	-0.3	0.83	0.48	0.01	0.17
0.2	-0.3	0.5	-0.2	-0.3	-0.3	0.82	0.49	-0.04	0.16
0.2	-0.3	0.6	-0.2	-0.3	-0.3	0.81	0.49	-0.10	0.14
0.2	-0.3	0.7	-0.2	-0.3	-0.3	0.80	0.50	-0.15	0.13
0.2	-0.3	0.8	-0.2	-0.3	-0.3	0.79	0.51	-0.21	0.11
0.2	-0.3	0.9	-0.2	-0.3	-0.3	0.78	0.51	-0.27	0.10
0.2	-0.3	1.0	-0.2	-0.3	-0.3	0.76	0.52	-0.33	0.08
0.3	-0.3	0.0	-0.4	-0.3	-0.3	0.87	0.46	0.21	0.22
0.3	-0.3	0.1	-0.4	-0.3	-0.3	0.81	0.44	0.21	0.21
0.3	-0.3	0.2	-0.4	-0.3	-0.3	0.79	0.45	0.16	0.20
0.3	-0.3	0.3	-0.4	-0.3	-0.3	0.77	0.46	0.10	0.19
0.3	-0.3	0.4	-0.4	-0.3	-0.3	0.76	0.47	0.05	0.18
0.3	-0.3	0.5	-0.4	-0.3	-0.3	0.74	0.48	-0.01	0.17
0.3	-0.3	0.6	-0.4	-0.3	-0.3	0.72	0.50	-0.07	0.15
0.3	-0.3	0.7	-0.4	-0.3	-0.3	0.70	0.51	-0.13	0.14
0.3	-0.3	0.8	-0.4	-0.3	-0.3	0.68	0.53	-0.20	0.12
0.3	-0.3	0.9	-0.4	-0.3	-0.3	0.66	0.54	-0.27	0.11
0.3	-0.3	1.0	-0.4	-0.3	-0.3	0.64	0.56	-0.35	0.09
0.3	-0.3	0.0	-0.5	-0.3	-0.3	0.81	0.42	0.28	0.23
0.3	-0.3	0.1	-0.5	-0.3	-0.3	0.79	0.43	0.23	0.22
0.3	-0.3	0.2	-0.5	-0.3	-0.3	0.77	0.44	0.18	0.21
0.3	-0.3	0.3	-0.5	-0.3	-0.3	0.75	0.45	0.13	0.20
<b>0.3</b>	<b>-0.3</b>	<b>0.4</b>	<b>-0.5</b>	<b>-0.3</b>	<b>-0.3</b>	0.73	<b>0.47</b>	<b>0.07</b>	<b>0.18</b>
0.3	-0.3	0.5	-0.5	-0.3	-0.3	0.71	0.48	0.01	0.17
0.3	-0.3	0.6	-0.5	-0.3	-0.3	0.70	0.49	-0.05	0.16
0.3	-0.3	0.7	-0.5	-0.3	-0.3	0.68	0.51	-0.11	0.14
0.3	-0.3	0.8	-0.5	-0.3	-0.3	0.66	0.52	-0.18	0.13
0.3	-0.3	0.9	-0.5	-0.3	-0.3	0.64	0.54	-0.25	0.11
0.3	-0.3	1.0	-0.5	-0.3	-0.3	0.62	0.55	-0.33	0.09
0.4	-0.3	0.0	-0.1	-0.3	-0.3	0.87	0.46	0.21	0.22
0.4	-0.3	0.1	-0.1	-0.3	-0.3	0.87	0.44	0.14	0.20
0.4	-0.3	0.2	-0.1	-0.3	-0.3	0.84	0.45	0.09	0.19
0.4	-0.3	0.3	-0.1	-0.3	-0.3	0.81	0.47	0.04	0.18
0.4	-0.3	0.4	-0.1	-0.3	-0.3	0.78	0.48	-0.01	0.17
0.4	-0.3	0.5	-0.1	-0.3	-0.3	0.75	0.50	-0.07	0.15
0.4	-0.3	0.6	-0.1	-0.3	-0.3	0.72	0.52	-0.13	0.14
0.4	-0.3	0.7	-0.1	-0.3	-0.3	0.69	0.55	-0.20	0.13
0.4	-0.3	0.8	-0.1	-0.3	-0.3	0.67	0.57	-0.27	0.11
0.4	-0.3	0.9	-0.1	-0.3	-0.3	0.64	0.59	-0.35	0.10
0.4	-0.3	1.0	-0.1	-0.3	-0.3	0.61	0.62	-0.44	0.08



## APPENDIX C. RESULTS OF REGRESSION ANALYSIS(A)

Regression analysis (A) as well as regression analysis (B) was conducted using various values for  $Y_i$  and  $P_i$ . In the following tables of outputs, the values used in runs are expressed in this table :

$Y_1$ Multiplier	0.15	$P_1$	1.00
$Y_2$ Multiplier	1.00	$P_2$	2.00
$Y_3$ Multiplier	0.15	$P_3$	0.50

This example means that  $Y_1$  and  $Y_3$  decreased by 50 percent and  $P_2$  and  $P_3$  became 2.0 and 0.5. Although these values change from the original data (Table 6) to the multiplied data which are provided by the next table, the multiplied data in each run are omitted in this Appendix and Appendix E for regression analysis (B).

Year	$G_1/P_1$	$G_2/P_2$	$G_3/P_3$	$Y_1$	$Y_2$	$Y_3$
1983	10.00	12.2191	2.7956	0.0964	0.0596	0.1091
1984	10.00	12.1896	2.7345	0.0961	0.0595	0.1069
1985	10.00	12.0399	2.6366	0.0959	0.0591	0.1025
1986	10.00	12.1775	2.6084	0.0964	0.0604	0.1011
1987	10.00	12.3153	2.5892	0.0944	0.0614	0.0994
1988	10.00	12.5444	2.5495	0.0904	0.0622	0.0946
1989	10.00	12.8663	2.4850	0.0862	0.0632	0.0875
1990	10.00	13.3214	2.3686	0.0808	0.0648	0.0776
1991	10.00	13.6538	1.7671	0.0772	0.0662	0.0607
1992	10.00	13.7121	1.1748	0.0732	0.0667	0.0434
1993	10.00	13.6135	0.6597	0.0731	0.0664	0.0312

1.

Y <sub>1</sub> Multiplier	1.00	P <sub>1</sub>	1.00
Y <sub>2</sub> Multiplier	1.00	P <sub>2</sub>	1.00
Y <sub>3</sub> Multiplier	1.00	P <sub>3</sub>	1.00

(7.A)

Multiple R	0.996					
R Square	0.992					
Adjusted R Square	0.865					
Standard Error	0.006					
	df	SS	MS	F	Significance F	
Regression	3	0.041	0.014	330	7E-08	
Residual	8	3E-04	4E-05			
Total	11	0.041				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>1</sub>	0.1536	0.008	18.66	7E-08	0.135	0.173
b <sub>2</sub> a <sub>1</sub> E <sub>2</sub>	-0.1558	0.011	-13.82	7E-07	-0.182	-0.130
b <sub>3</sub> a <sub>1</sub> E <sub>3</sub>	0.0094	0.003	3.532	8E-03	0.003	0.016

(7.B)

Multiple R	0.989					
R Square	0.978					
Adjusted R Square	0.848					
Standard Error	5E-04					
	df	SS	MS	F	Significance F	
Regression	3	8E-05	3E-05	119	2E-06	
Residual	8	2E-06	2E-07			
Total	11	9E-05				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>1</sub> a <sub>2</sub> E <sub>1</sub>	0.0009	0.001	1.455	0.184	-0.001	0.002
b <sub>2</sub>	0.0084	0.001	9.804	1E-05	0.006	0.010
b <sub>3</sub> a <sub>2</sub> E <sub>3</sub>	-0.0001	2E-04	-0.395	0.703	-0.001	4E-04

(7.C)

Multiple R	0.998					
R Square	0.996					
Adjusted R Square	0.871					
Standard Error	0.012					
	df	SS	MS	F	Significance F	
Regression	3	0.318	0.106	748	4E-09	
Residual	8	0.001	1E-04			
Total	11	0.319				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>1</sub> a <sub>3</sub> E <sub>1</sub>	0.1416	0.015	9.262	2E-05	0.106	0.177
b <sub>2</sub> a <sub>3</sub> E <sub>2</sub>	-0.1938	0.021	-9.254	2E-05	-0.242	-0.146
b <sub>3</sub>	0.0850	0.005	17.15	1E-07	0.074	0.096

b <sub>1</sub>	0.1536	b <sub>2</sub> a <sub>1</sub> E <sub>2</sub>	0.1558	E <sub>2</sub>	21.8413
a <sub>1</sub>	0.8464	b <sub>3</sub> a <sub>1</sub> E <sub>3</sub>	-0.0094	E <sub>3</sub>	-0.1310
b <sub>2</sub>	0.0084	b <sub>1</sub> a <sub>2</sub> E <sub>1</sub>	-0.0009	E <sub>1</sub>	-0.0060
a <sub>2</sub>	0.9916	b <sub>3</sub> a <sub>2</sub> E <sub>3</sub>	0.0001	E <sub>3</sub>	0.0010
b <sub>3</sub>	0.0850	b <sub>1</sub> a <sub>3</sub> E <sub>1</sub>	-0.1416	E <sub>1</sub>	-1.0076
b <sub>3</sub>	0.9150	b <sub>2</sub> a <sub>3</sub> E <sub>2</sub>	0.1938	E <sub>2</sub>	25.1270

2.

Y <sub>1</sub> Multiplier	0.20	P <sub>1</sub>	1.00
Y <sub>2</sub> Multiplier	1.00	P <sub>2</sub>	1.00
Y <sub>3</sub> Multiplier	0.20	P <sub>3</sub>	1.00

(7.A)

Multiple R	0.996					
R Square	0.992					
Adjusted R Square	0.865					
Standard Error	0.001					
	df	SS	MS	F	Significance F	
Regression	3	0.002	0.001	330	7E-08	
Residual	8	1E-05	2E-06			
Total	11	0.002				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>1</sub>	0.0307	0.002	18.66	7E-08	0.027	0.035
b <sub>2</sub> a <sub>1</sub> E <sub>2</sub>	-0.0312	0.002	-13.82	7E-07	-0.036	-0.026
b <sub>1</sub> a <sub>1</sub> E <sub>3</sub>	0.0019	0.001	3.532	0.008	0.001	0.003

(7.B)

Multiple R	0.989					
R Square	0.978					
Adjusted R Square	0.848					
Standard Error	5E-04					
	df	SS	MS	F	Significance F	
Regression	3	8E-05	3E-05	119	2E-06	
Residual	8	2E-06	2E-07			
Total	11	9E-05				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>1</sub> a <sub>2</sub> E <sub>1</sub>	0.0009	0.001	1.455	0.184	-0.001	0.002
b <sub>2</sub>	0.0084	0.001	9.804	1E-05	0.006	0.010
b <sub>3</sub> a <sub>2</sub> E <sub>3</sub>	-0.0001	2E-04	-0.395	0.703	-0.001	4E-04

(7.C)

Multiple R	0.998					
R Square	0.996					
Adjusted R Square	0.871					
Standard Error	0.002					
	df	SS	MS	F	Significance F	
Regression	3	0.013	0.004	748	4E-09	
Residual	8	5E-05	6E-06			
Total	11	0.013				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>1</sub> a <sub>3</sub> E <sub>1</sub>	0.0283	0.003	9.262	2E-05	0.021	0.035
b <sub>2</sub> a <sub>3</sub> E <sub>2</sub>	-0.0388	0.004	-9.254	2E-05	-0.048	-0.029
b <sub>3</sub>	0.0170	0.001	17.15	1E-07	0.015	0.019

b <sub>1</sub>	0.0307	b <sub>2</sub> a <sub>1</sub> E <sub>2</sub>	0.0312	E <sub>2</sub>	3.8143
a <sub>1</sub>	0.9693	b <sub>3</sub> a <sub>1</sub> E <sub>3</sub>	-0.0019	E <sub>3</sub>	-0.1144
b <sub>2</sub>	0.0084	b <sub>1</sub> a <sub>2</sub> E <sub>1</sub>	-0.0009	E <sub>1</sub>	-0.0300
a <sub>2</sub>	0.9916	b <sub>3</sub> a <sub>2</sub> E <sub>3</sub>	0.0001	E <sub>3</sub>	0.0048
b <sub>3</sub>	0.0170	b <sub>1</sub> a <sub>3</sub> E <sub>1</sub>	-0.0283	E <sub>1</sub>	-0.9379
b <sub>3</sub>	0.9830	b <sub>2</sub> a <sub>3</sub> E <sub>2</sub>	0.0388	E <sub>2</sub>	4.6777

3.

Y <sub>1</sub> Multiplier	0.15	P <sub>1</sub>	1.00
Y <sub>2</sub> Multiplier	1.00	P <sub>2</sub>	1.00
Y <sub>3</sub> Multiplier	0.15	P <sub>3</sub>	1.00

(7.A)

7.2.7

Multiple R	0.996
R Square	0.992
Adjusted R Square	0.865
Standard Error	0.001

	df	SS	MS	F	Significance F
Regression	3	0.001	3E-04	330	7E-08
Residual	8	7E-06	9E-07		
Total	11	0.001			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>1</sub>	0.0230	0.001	18.66	7E-08	0.020	0.026
b <sub>2a</sub> E <sub>2</sub>	-0.0234	0.002	-13.82	7E-07	-0.027	-0.019
b <sub>3a</sub> E <sub>3</sub>	0.0014	4E-04	3.532	0.008	5E-04	0.002

(7.B)

Multiple R	0.989					
R Square	0.978					
Adjusted R Square	0.848					
Standard Error	5E-04					
	df	SS	MS	F	Significance F	
Regression	3	8E-05	3E-05	119	2E-06	
Residual	8	2E-06	2E-07			
Total	11	9E-05				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>1a</sub> E <sub>21</sub>	0.0009	0.001	1.455	0.184	-0.001	0.002
b <sub>2</sub>	0.0084	0.001	9.804	1E-05	0.006	0.010
b <sub>3a</sub> E <sub>23</sub>	-8E-05	2E-04	-0.395	0.703	-0.001	4E-04

(7.C)

(7C)

Multiple R	0.998
R Square	0.996
Adjusted R Square	0.871
Standard Error	0.002

	df	SS	MS	F	Significance F
Regression	3	0.007	0.002	748	4E-09
Residual	8	3E-05	3E-06		
Total	11	0.007			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>1a</sub> E <sub>31</sub>	0.0212	0.002	9.262	2E-05	0.016	0.027
b <sub>2a</sub> E <sub>32</sub>	-0.0291	0.003	-9.254	2E-05	-0.036	-0.022
b <sub>3</sub>	0.0128	0.001	17.15	1E-07	0.011	0.014

b <sub>1</sub>	0.0230	b <sub>2a</sub> E <sub>12</sub>	0.0234	E <sub>2</sub>	2.8382
a <sub>1</sub>	0.9770	b <sub>3a</sub> E <sub>13</sub>	-0.0014	E <sub>3</sub>	-0.1135
b <sub>2</sub>	0.0084	b <sub>1a</sub> E <sub>21</sub>	-0.0009	E <sub>1</sub>	-0.0400
a <sub>2</sub>	0.9916	b <sub>3a</sub> E <sub>23</sub>	0.0001	E <sub>3</sub>	0.0064
b <sub>3</sub>	0.0128	b <sub>1a</sub> E <sub>31</sub>	-0.0212	E <sub>1</sub>	-0.9338
b <sub>3</sub>	0.9872	b <sub>2a</sub> E <sub>32</sub>	0.0291	E <sub>2</sub>	3.4932

4.

Y <sub>1</sub> Multiplier	0.10	P <sub>1</sub>	1.00
Y <sub>2</sub> Multiplier	1.00	P <sub>2</sub>	1.00
Y <sub>3</sub> Multiplier	0.10	P <sub>3</sub>	1.00

(7.A)

Multiple R	0.996					
R Square	0.992					
Adjusted R Square	0.865					
Standard Error	0.001					
	df	SS	MS	F	Significance F	
Regression	3	4E-04	1E-04	330	7E-08	
Residual	8	3E-06	4E-07			
Total	11	4E-04				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>1</sub>	0.0154	0.001	18.66	7E-08	0.013	0.017
b <sub>2</sub> a <sub>1</sub> E <sub>2</sub>	-0.0156	0.001	-13.82	7E-07	-0.018	-0.013
b <sub>3</sub> a <sub>1</sub> E <sub>3</sub>	0.0009	3E-04	3.532	0.008	3E-04	0.002

(7.B)

Multiple R	0.989					
R Square	0.978					
Adjusted R Square	0.848					
Standard Error	5E-04					
	df	SS	MS	F	Significance F	
Regression	3	8E-05	3E-05	119	2E-06	
Residual	8	2E-06	2E-07			
Total	11	9E-05				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>1</sub> a <sub>2</sub> E <sub>2</sub>	0.0009	0.001	1.455	0.184	-0.001	0.002
b <sub>2</sub>	0.0084	0.001	9.804	1E-05	0.006	0.010
b <sub>3</sub> a <sub>2</sub> E <sub>3</sub>	-0.0001	2E-04	-0.395	0.703	-0.001	4E-04

(7.C)

Multiple R	0.998					
R Square	0.996					
Adjusted R Square	0.871					
Standard Error	0.001					
	df	SS	MS	F	Significance F	
Regression	3	0.003	0.001	748	4E-09	
Residual	8	1E-05	1E-06			
Total	11	0.003				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>1</sub> a <sub>3</sub> E <sub>3</sub>	0.0142	0.002	9.262	2E-05	0.011	0.018
b <sub>2</sub> a <sub>3</sub> E <sub>2</sub>	-0.0194	0.002	-9.254	2E-05	-0.024	-0.015
b <sub>3</sub>	0.0085	5E-04	17.15	1E-07	0.007	0.010

b <sub>1</sub>	0.0154	b <sub>2</sub> a <sub>1</sub> E <sub>2</sub>	0.0156	E <sub>2</sub>	1.8774
a <sub>1</sub>	0.9846	b <sub>3</sub> a <sub>1</sub> E <sub>3</sub>	-0.0009	E <sub>3</sub>	-0.1126
b <sub>2</sub>	0.0084	b <sub>1</sub> a <sub>2</sub> E <sub>2</sub>	-0.0009	E <sub>2</sub>	-0.0600
a <sub>2</sub>	0.9916	b <sub>3</sub> a <sub>2</sub> E <sub>3</sub>	0.0001	E <sub>3</sub>	0.0095
b <sub>3</sub>	0.0085	b <sub>1</sub> a <sub>3</sub> E <sub>3</sub>	-0.0142	E <sub>3</sub>	-0.9298
a <sub>3</sub>	0.9915	b <sub>2</sub> a <sub>3</sub> E <sub>2</sub>	0.0194	E <sub>2</sub>	2.3188

5.

Y <sub>1</sub> Multiplier	0.15	P <sub>1</sub>	1.00
Y <sub>2</sub> Multiplier	1.00	P <sub>2</sub>	1.50
Y <sub>3</sub> Multiplier	0.15	P <sub>3</sub>	0.75

(7.A)

Multiple R	0.996					
R Square	0.992					
Adjusted R Square	0.865					
Standard Error	0.001					
	df	SS	MS	F	Significance F	
Regression	3	0.001	3E-04	330	7E-08	
Residual	8	7E-06	9E-07			
Total	11	0.001				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>1</sub>	0.0230	0.001	18.66	7E-08	0.020	0.026
b <sub>2</sub> a <sub>1</sub> E <sub>2</sub>	-0.0156	0.001	-13.82	7E-07	-0.018	-0.013
b <sub>3</sub> a <sub>1</sub> E <sub>3</sub>	0.0019	0.001	3.532	0.008	0.001	0.003

(7.B)

Multiple R	0.989					
R Square	0.978					
Adjusted R Square	0.848					
Standard Error	5E-04					
	df	SS	MS	F	Significance F	
Regression	3	8E-05	3E-05	119	2E-06	
Residual	8	2E-06	2E-07			
Total	11	9E-05				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>1</sub> a <sub>2</sub> E <sub>21</sub>	0.0009	0.001	1.455	0.184	-0.001	0.002
b <sub>2</sub>	0.0056	0.001	9.804	1E-05	0.004	0.007
b <sub>3</sub> a <sub>2</sub> E <sub>3</sub>	-0.0001	3E-04	-0.395	0.703	-0.001	0.001

(7.C)

Multiple R	0.998					
R Square	0.996					
Adjusted R Square	0.871					
Standard Error	0.002					
	df	SS	MS	F	Significance F	
Regression	3	0.007	0.002	748	4E-09	
Residual	8	3E-05	3E-06			
Total	11	0.007				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
$b_1$ $a_3 E_1$	0.0212	0.002	9.262	2E-05	0.016	0.027
$b_2$ $a_3 E_2$	-0.0194	0.002	-9.254	2E-05	-0.024	-0.015
$b_3$	0.0170	0.001	17.15	1E-07	0.015	0.019

b <sub>1</sub>	0.0230	b <sub>2</sub> a <sub>1</sub> E <sub>2</sub>	0.0156	E <sub>2</sub>	2.8382
a <sub>1</sub>	0.9770	b <sub>3</sub> a <sub>1</sub> E <sub>3</sub>	-0.0019	E <sub>3</sub>	-0.1135
b <sub>2</sub>	0.0056	b <sub>1</sub> a <sub>2</sub> E <sub>1</sub>	-0.0009	E <sub>1</sub>	-0.0399
a <sub>2</sub>	0.9944	b <sub>3</sub> a <sub>2</sub> E <sub>3</sub>	0.0001	E <sub>3</sub>	0.0063
b <sub>3</sub>	0.0170	b <sub>1</sub> a <sub>3</sub> E <sub>1</sub>	-0.0212	E <sub>1</sub>	-0.9379
b <sub>3</sub>	0.9830	b <sub>2</sub> a <sub>3</sub> E <sub>2</sub>	0.0194	E <sub>2</sub>	3.5083

6.

$Y_1$ Multiplier	0.15	$P_1$	1.00
$Y_2$ Multiplier	1.00	$P_2$	2.00
$Y_3$ Multiplier	0.15	$P_3$	0.50

(7.A)

Multiple R	0.996					
R Square	0.992					
Adjusted R Square	0.865					
Standard Error	0.001					
	df	SS	MS	F	Significance F	
Regression	3	0.001	3E-04	330	7E-08	
Residual	8	7E-06	9E-07			
Total	11	0.001				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>1</sub>	0.0230	0.001	18.66	7E-08	0.020	0.026
b <sub>2</sub> a <sub>1</sub> E <sub>2</sub>	-0.0117	0.001	-13.82	7E-07	-0.014	-0.010
b <sub>3</sub> a <sub>1</sub> E <sub>3</sub>	0.0028	0.001	3.532	0.008	0.001	0.005

(7.B)

Multiple R	0.989					
R Square	0.978					
Adjusted R Square	0.848					
Standard Error	5E-04					
	df	SS	MS	F	Significance F	
Regression	3	8E-05	3E-05	119	2E-06	
Residual	8	2E-06	2E-07			
Total	11	9E-05				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>1</sub> a <sub>2</sub> E <sub>1</sub>	0.0009	0.001	1.455	0.184	-0.001	0.002
b <sub>2</sub>	0.0042	4E-04	9.804	1E-05	0.003	0.005
b <sub>3</sub> a <sub>2</sub> E <sub>3</sub>	-0.0002	4E-04	-0.395	0.703	-0.001	0.001

(7.C)

Multiple R	0.998					
R Square	0.996					
Adjusted R Square	0.871					
Standard Error	0.002					
	df	SS	MS	F	Significance F	
Regression	3	0.007	0.002	748	4E-09	
Residual	8	3E-05	3E-06			
Total	11	0.007				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
$b_1$ $a_3 E_1$	0.0212	0.002	9.262	2E-05	0.016	0.027
$b_2$ $a_3 E_2$	-0.0145	0.002	-9.254	2E-05	-0.018	-0.011
$b_3$	0.0255	0.001	17.15	1E-07	0.022	0.029

$b_1$	0.0230	$b_2, a_1 E_2$	0.0117	$E_2$	2.8382
$a_1$	0.9770	$b_1, a_1 E_3$	-0.0028	$E_3$	-0.1135
$b_2$	0.0042	$b_1, a_2 E_1$	-0.0009	$E_1$	-0.0398
$a_2$	0.9958	$b_3, a_2 E_3$	0.0002	$E_3$	0.0063
$b_3$	0.0255	$b_1, a_3 E_1$	-0.0212	$E_1$	-0.9460
$b_3$	0.9745	$b_2, a_2 E_2$	0.0145	$E_2$	3.5389

7.

Y <sub>1</sub> Multiplier	0.15	P <sub>1</sub>	1.00
Y <sub>2</sub> Multiplier	1.00	P <sub>2</sub>	0.50
Y <sub>3</sub> Multiplier	0.15	P <sub>3</sub>	2.00

(7.A)

Multiple R	0.996					
R Square	0.992					
Adjusted R Square	0.865					
Standard Error	0.001					
	df	SS	MS	F	Significance F	
Regression	3	0.001	3E-04	330	7E-08	
Residual	8	7E-06	9E-07			
Total	11	0.001				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>1</sub>	0.0230	0.001	18.66	7E-08	0.020	0.026
b <sub>2</sub> aE <sub>1,2</sub>	-0.0468	0.003	-13.82	7E-07	-0.055	-0.039
b <sub>3</sub> aE <sub>1,1</sub>	0.0007	2E-04	3.532	0.008	2E-04	0.001

(7.B)

Multiple R	0.989					
R Square	0.978					
Adjusted R Square	0.848					
Standard Error	5E-04					
	df	SS	MS	F	Significance F	
Regression	3	8E-05	3E-05	119	2E-06	
Residual	8	2E-06	2E-07			
Total	11	9E-05				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>1</sub> a E <sub>2,1</sub>	0.0009	0.001	1.455	0.184	-0.001	0.002
b <sub>2</sub>	0.0169	0.002	9.804	1E-05	0.013	0.021
b <sub>3</sub> a E <sub>2,1</sub>	-4E-05	1E-04	-0.395	0.703	-3E-04	2E-04

(7.C)

Multiple R	0.998					
R Square	0.996					
Adjusted R Square	0.871					
Standard Error	0.002					
	df	SS	MS	F	Significance F	
Regression	3	0.007	0.002	748	4E-09	
Residual	8	3E-05	3E-06			
Total	11	0.007				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>1</sub> a E <sub>1</sub>	0.0212	0.002	9.262	2E-05	0.016	0.027
b <sub>2</sub> a E <sub>2</sub>	-0.0581	0.006	-9.254	2E-05	-0.073	-0.044
b <sub>3</sub>	0.0064	4E-04	17.15	1E-07	0.006	0.007

b <sub>1</sub>	0.0230	b <sub>1</sub> a E <sub>1,1</sub>	0.0468	E <sub>1,1</sub>	2.8382
a <sub>1</sub>	0.9770	b <sub>1</sub> a E <sub>2,1</sub>	-0.0007	E <sub>2,1</sub>	-0.1135
b <sub>2</sub>	0.0169	b <sub>1</sub> a E <sub>3,1</sub>	-0.0009	E <sub>3,1</sub>	-0.0403
a <sub>2</sub>	0.9831	b <sub>2</sub> a E <sub>1,1</sub>	0.0000	E <sub>1,1</sub>	0.0064
b <sub>3</sub>	0.0064	b <sub>2</sub> a E <sub>2,1</sub>	-0.0212	E <sub>2,1</sub>	-0.9278
b <sub>3</sub>	0.9936	b <sub>2</sub> a E <sub>3,1</sub>	0.0581	E <sub>3,1</sub>	3.4708

## APPENDIX D. CALCULATION OF A LEADER - FOLLOWER FRAMEWORK

### 1. FOLLOWERS

Two reaction functions for Followers are provided by the equations (8.A) and (8.B).

$$Y_2 = (b_2) \left( \frac{G_2}{P_2} \right) - (a_2 E_{21}) Y_1 - (a_2 E_{23}) Y_3$$

$$Y_3 = (b_3) \left( \frac{G_3}{P_3} \right) - (a_3 E_{31}) Y_1 - (a_3 E_{32}) Y_2$$

Substituting (8.B) into (8.A), a function of  $Y_2$  in terms of  $Y_1$  is obtained as (9.A) :

$$\begin{aligned} Y_2 &= (b_2) \left( \frac{G_2}{P_2} \right) - (a_2 E_{21}) Y_1 - (a_2 E_{23}) \left[ (b_3) \left( \frac{G_3}{P_3} \right) - (a_3 E_{31}) Y_1 - (a_3 E_{32}) Y_2 \right] \\ &= b_2 \frac{G_2}{P_2} - a_2 E_{21} Y_1 - a_2 E_{23} b_3 \frac{G_3}{P_3} + a_2 E_{23} a_3 E_{31} Y_1 + a_2 E_{23} a_3 E_{32} Y_2 \end{aligned}$$

$$\begin{aligned} Y_2 - a_2 E_{23} a_3 E_{32} Y_2 &= b_2 \frac{G_2}{P_2} - a_2 E_{21} Y_1 + a_2 E_{23} a_3 E_{31} Y_1 - a_2 E_{23} b_3 \frac{G_3}{P_3} \\ (1 - a_2 E_{23} a_3 E_{32}) Y_2 &= b_2 \frac{G_2}{P_2} + (a_2 E_{23} a_3 E_{31} - a_2 E_{21}) Y_1 - a_2 E_{23} b_3 \frac{G_3}{P_3} \end{aligned}$$

$$Y_2 = \frac{(b_2) \frac{G_2}{P_2} - (a_2 E_{21} - a_2 a_3 E_{23} E_{31}) Y_1 - (b_3 a_2 E_{23}) \frac{G_3}{P_3}}{1 - a_2 a_3 E_{23} E_{32}}$$

Substituting (9.A) into (8.B), a function of  $Y_3$  in terms of  $Y_1$  becomes (9.B).

$$\begin{aligned}
Y_3 &= (b_3) \left( \frac{G_3}{P_3} \right) - (a_3 E_{31}) Y_1 \\
&\quad - (a_3 E_{32}) \left[ \frac{b_2 \frac{G_2}{P_2} + (a_2 a_3 E_{23} E_{31} - a_2 E_{21}) Y_1 - a_2 b_3 E_{23} \frac{G_3}{P_3}}{1 - a_2 a_3 E_{23} E_{32}} \right] \\
Y_3 &= \left( b_3 + \frac{a_2 a_3 b_3 E_{23} E_{32}}{1 - a_2 a_3 E_{23} E_{32}} \right) \frac{G_3}{P_3} \\
&\quad - \left[ (a_3 E_{32}) \frac{a_2 a_3 E_{23} E_{31} - a_2 E_{21}}{1 - a_2 a_3 E_{23} E_{32}} - a_3 E_{31} \right] Y_1 - \left( \frac{a_3 b_2 E_{32}}{1 - a_2 a_3 E_{23} E_{32}} \right) \frac{G_2}{P_2} \\
Y_3 &= \frac{(b_3) \frac{G_3}{P_3} - (a_3 E_{31} - a_2 a_3 E_{21} E_{32}) Y_1 - (b_2 a_3 E_{32}) \frac{G_2}{P_2}}{1 - a_2 a_3 E_{23} E_{32}}
\end{aligned}$$

## 2. A LEADER

To get a Leader's model, the utility function for  $Y_1$  is maximized.

Maximizing :  $U_1 = X_1^{A_1} Z_1^{B_1}$

subject to  $G_1 - X_1 - P_1 Y_1 = 0$

where  $Z_1 = Y_1 + E_{12} Y_2 + E_{13} Y_3$

The Lagrangean-function for this maximizing becomes :

$$L = X_1^{A_1} (Y_1 + E_{12} Y_2 + E_{13} Y_3)^{B_1} + \lambda (G_1 - X_1 - P_1 Y_1)$$

By differentiation with respect to  $X_1, Y_1$  and  $\lambda$ , three equations are prepared :

$$\begin{aligned}
\textcircled{1} \quad & L_{X_1} = Z_1^{B_1} A_1 (X_1)^{A_1-1} - \lambda = 0 \\
\textcircled{2} \quad & L_{Y_1} = X_1^{A_1} B_1 (Z_1)^{B_1-1} \left( \frac{dZ_1}{dY_1} \right) - \lambda P_1 = 0 \\
\textcircled{3} \quad & L_\lambda = G_1 - X_1 - P_1 Y_1 = 0
\end{aligned}$$

Dividing ① by ②, results in :

$$\begin{aligned}
\frac{L_{X_1}}{L_{Y_1}} &= \frac{Z_1^{B_1} A_1 X_1^{A_1-1}}{X_1^{A_1} B_1 Z_1^{B_1-1} \left( \frac{dZ_1}{dY_1} \right)} = \frac{\lambda}{\lambda P_1} \\
&= \frac{A_1 Z_1}{B_1 X_1 \left( \frac{dZ_1}{dY_1} \right)} = \frac{1}{P_1}
\end{aligned}$$

$$\textcircled{4} \quad X_1 = \frac{P_1 A_1 Z_1}{B_1 \left( \frac{dZ_1}{dY_1} \right)}$$

③ is transformed by ④.

$$\begin{aligned}
\textcircled{5} \quad Y_1 &= \frac{G_1 - X_1}{P_1} = \frac{G_1}{P_1} - \frac{A_1 Z_1}{B_1 \left( \frac{dZ_1}{dY_1} \right)} \\
&= \frac{G_1}{P_1} - \frac{A_1 (Y_1 + E_{12} Y_2 + E_{13} Y_3)}{B_1 \left( 1 + E_{12} \frac{dY_2}{dY_1} + E_{13} \frac{dY_3}{dY_1} \right)}
\end{aligned}$$

where  $Z_1 = Y_1 + E_{12} Y_2 + E_{13} Y_3$ .

Because functions of  $Y_2$  and  $Y_3$  as (9.A) and (9.B) have been derived, the following may be calculated :

$$\textcircled{6} \quad \frac{dY_2}{dY_1} = \frac{-a_2 E_{21} + a_2 a_3 E_{23} E_{31}}{1 - a_2 a_3 E_{23} E_{32}}$$

$$\textcircled{7} \quad \frac{dY_2}{dY_1} = \frac{-a_3 E_{31} + a_2 a_3 E_{21} E_{32}}{1 - a_2 a_3 E_{23} E_{32}}$$

Substitute (9.A), (9.B),  $\textcircled{6}$  and  $\textcircled{7}$  into  $\textcircled{5}$ ,

$$Y_1 = \frac{G_1}{P_1} - \frac{A_1 \left\{ \begin{aligned} & \left[ Y_1 + E_{12} \right] \left[ \frac{(b_2) \frac{G_2}{P_2} - (a_2 E_{21} + a_2 a_3 E_{23} E_{31}) Y_1 - (a_2 b_3 E_{23}) \frac{G_3}{P_3}}{1 - a_2 a_3 E_{23} E_{32}} \right] \\ & + E_{13} \left[ \frac{(b_3) \frac{G_3}{P_3} - (a_3 E_{31} + a_2 a_3 E_{21} E_{32}) Y_1 - (a_3 b_2 E_{32}) \frac{G_2}{P_2}}{1 - a_2 a_3 E_{23} E_{32}} \right] \end{aligned} \right\}}{B_1 \left[ 1 - E_{12} \left( \frac{-a_2 E_{21} + a_2 a_3 E_{23} E_{31}}{1 - a_2 a_3 E_{23} E_{32}} \right) - E_{13} \left( \frac{-a_3 E_{31} + a_2 a_3 E_{21} E_{32}}{1 - a_2 a_3 E_{23} E_{32}} \right) \right]}$$

The result is :

$$Y_1 \left\{ 1 + \frac{A_1 \left[ 1 - \frac{E_{12}(a_2 E_{21} - a_2 a_3 E_{23} E_{31}) + E_{13}(a_3 E_{31} - a_2 a_3 E_{21} E_{32})}{1 - a_2 a_3 E_{23} E_{32}} \right]}{B_1 \left[ 1 - \frac{E_{12}(a_2 E_{21} - a_2 a_3 E_{23} E_{31}) + E_{13}(a_3 E_{31} - a_2 a_3 E_{21} E_{32})}{1 - a_2 a_3 E_{23} E_{32}} \right]} \right\} =$$

$$\frac{G_1}{P_1} - \frac{A_1 \left[ \frac{(b_2 E_{12} - a_3 b_2 E_{13} E_{32}) \frac{G_2}{P_2} + (b_3 E_{13} - a_2 b_3 E_{12} E_{23}) \frac{G_3}{P_3}}{1 - a_2 a_3 E_{23} E_{32}} \right]}{B_1 \left[ 1 - \frac{E_{12}(a_2 E_{21} - a_2 a_3 E_{23} E_{31}) + E_{13}(a_3 E_{31} - a_2 a_3 E_{21} E_{32})}{1 - a_2 a_3 E_{23} E_{32}} \right]}$$

$$Y_1 \left( \frac{B_1 + A_1}{B_1} \right) = \frac{G_1}{P_1} - \frac{A_1 \left[ (b_2 E_{12} - a_3 b_2 E_{13} E_{32}) \frac{G_2}{P_2} + (b_3 E_{13} - a_2 b_3 E_{12} E_{23}) \frac{G_3}{P_3} \right]}{B_1 \left( \begin{array}{l} 1 - a_2 a_3 E_{23} E_{32} - a_2 E_{12} E_{21} + a_2 a_3 E_{12} E_{23} E_{31} \\ - a_3 E_{13} E_{31} + a_2 a_3 E_{13} E_{21} E_{32} \end{array} \right)}$$

Finally, I obtain the equation (11) as :

$$Y_1 = (b_1) \frac{G_1}{P_1} - (b_1) \frac{\left[ (A_1 b_2 E_{12} - A_1 a_3 b_2 E_{13} E_{32}) \frac{G_2}{P_2} + (A_1 b_3 E_{13} - A_1 a_2 b_3 E_{12} E_{23}) \frac{G_3}{P_3} \right]}{\left( \begin{array}{l} B_1 - B_1 a_2 a_3 E_{23} E_{32} - B_1 a_2 E_{12} E_{21} + B_1 a_2 a_3 E_{12} E_{23} E_{31} \\ - B_1 a_3 E_{13} E_{31} + B_1 a_2 a_3 E_{13} E_{21} E_{32} \end{array} \right)}$$

where  $\frac{B_1}{A_1 + B_1} = b_1$



## APPENDIX E. RESULTS OF REGRESSION ANALYSIS(B)

1.

Y <sub>1</sub> Multiplier	1.00	P <sub>1</sub>	1.00
Y <sub>2</sub> Multiplier	1.00	P <sub>2</sub>	1.00
Y <sub>3</sub> Multiplier	1.00	P <sub>3</sub>	1.00

(12.A)

Multiple R	0.996					
R Square	0.992					
Adjusted R Square	0.865					
Standard Error	0.006					
	df	SS	MS	F	Significance F	
Regression	3	0.041	0.014	330	7E-08	
Residual	8	3E-04	4E-05			
Total	11	0.041				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>1</sub>	0.15364	0.008	18.7	7E-08	0.135	0.173
b <sub>2</sub> aE <sub>12</sub>	-0.15584	0.011	-13.8	7E-07	-0.182	-0.130
b <sub>3</sub> aE <sub>13</sub>	0.00943	0.003	3.532	0.008	0.003	0.016

(12.B)

(12.8)

Multiple R	0.989					
R Square	0.978					
Adjusted R Square	0.847					
Standard Error	5E-04					
	df	SS	MS	F	Significance F	
Regression	3	8E-05	3E-05	117	2E-06	
Residual	8	2E-06	2E-07			
Total	11	9E-05				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>2</sub>	0.0094	2E-04	40.62	1E-10	0.009	0.010
b <sub>3</sub> aE <sub>23</sub>	-0.0001	2E-04	-0.510	0.624	-0.001	4E-04
a <sub>2</sub> E <sub>1</sub>	0.0057	0.004	1.389	0.202	-0.004	0.015

(12.C)

(12C)						
Multiple R	0.999					
R Square	0.998					
Adjusted R Square	0.873					
Standard Error	0.008					
	df	SS	MS	F	Significance F	
Regression	3	0.318	0.106	1498	3E-10	
Residual	8	0.001	7E-05			
Total	11	0.319				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>2</sub> aE <sub>2</sub>	-0.0507	0.004	-12.9	1E-06	-0.060	-0.042
b <sub>3</sub>	0.0758	0.004	18.6	7E-08	0.066	0.085
aE <sub>1</sub>	0.9325	0.070	13.4	9E-07	0.772	1.093

b <sub>1</sub>	0.1536	b <sub>2</sub> aE <sub>2</sub>	0.1558	E12	19.6471
a <sub>1</sub>	0.8464	b <sub>3</sub> aE <sub>3</sub>	-0.0094	E13	-0.1470
b <sub>2</sub>	0.0094	a <sub>2</sub> E <sub>1</sub>	-0.0057	E21	-0.0057
a <sub>2</sub>	0.9906	b <sub>3</sub> aE <sub>3</sub>	0.0001	E23	0.0016
b <sub>3</sub>	0.0758	a <sub>3</sub> E <sub>1</sub>	-0.9325	E31	-1.0089
a <sub>3</sub>	0.9242	b <sub>2</sub> aE <sub>2</sub>	0.0507	E32	5.8557

2.

Y <sub>1</sub> Multiplier	0.20	P <sub>1</sub>	1.00
Y <sub>2</sub> Multiplier	1.00	P <sub>2</sub>	1.00
Y <sub>3</sub> Multiplier	0.20	P <sub>3</sub>	1.00

(12.A)

Multiple R	0.996					
R Square	0.992					
Adjusted R Square	0.865					
Standard Error	0.001					
	df	SS	MS	F	Significance F	
Regression	3	0.002	0.001	330	7E-08	
Residual	8	1E-05	2E-06			
Total	11	0.002				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>1</sub>	0.0307	0.002	18.66	7E-08	0.027	0.035
b <sub>2</sub> aE <sub>12</sub>	-0.0312	0.002	-13.82	7E-07	-0.036	-0.026
b <sub>3</sub> aE <sub>13</sub>	0.0019	0.001	3.532	0.008	0.001	0.003

(12.B)

Multiple R	0.989					
R Square	0.978					
Adjusted R Square	0.847					
Standard Error	5E-04					
	df	SS	MS	F	Significance F	
Regression	3	8E-05	3E-05	117	2E-06	
Residual	8	2E-06	2E-07			
Total	11	9E-05				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>2</sub>	0.0094	2E-04	40.62	1E-10	0.009	0.010
b <sub>3</sub> aE <sub>23</sub>	-0.0001	2E-04	-0.510	0.624	-0.001	4E-04
a <sub>2</sub> E <sub>1</sub>	0.0283	0.020	1.389	0.202	-0.019	0.075

(12.C)

Multiple R	0.999
R Square	0.998
Adjusted R Square	0.873
Standard Error	0.002

	df	SS	MS	F	Significance F
Regression	3	0.013	0.004	1498	3E-10
Residual	8	2E-05	3E-06		
Total	11	0.013			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>2</sub> aE <sub>12</sub>	-0.0101	0.001	-12.88	1E-06	-0.012	-0.008
b <sub>3</sub>	0.0152	0.001	18.63	7E-08	0.013	0.017
a <sub>2</sub> E <sub>1</sub>	0.9325	0.070	13.39	9E-07	0.772	1.093

b <sub>1</sub>	0.0307	b <sub>2</sub> aE <sub>12</sub>	0.0312	E12	3.4311
a <sub>1</sub>	0.9693	b <sub>3</sub> aE <sub>13</sub>	-0.0019	E13	-0.1284
b <sub>2</sub>	0.0094	a <sub>2</sub> E <sub>1</sub>	-0.0283	E21	-0.0286
a <sub>2</sub>	0.9906	b <sub>1</sub> aE <sub>11</sub>	0.0001	E23	0.0081
b <sub>3</sub>	0.0152	a <sub>3</sub> E <sub>1</sub>	-0.9325	E31	-0.9468
a <sub>3</sub>	0.9848	b <sub>2</sub> aE <sub>12</sub>	0.0101	E32	1.0991

3.

Y <sub>1</sub> Multiplier	0.15	P <sub>1</sub>	1.00
Y <sub>2</sub> Multiplier	1.00	P <sub>2</sub>	1.00
Y <sub>3</sub> Multiplier	0.15	P <sub>3</sub>	1.00

(12.A)

Multiple R	0.996					
R Square	0.992					
Adjusted R Square	0.865					
Standard Error	0.001					
	df	SS	MS	F	Significance F	
Regression	3	0.001	3E-04	330	7E-08	
Residual	8	7E-06	9E-07			
Total	11	0.001				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>1</sub>	0.0230	0.001	18.7	7E-08	0.020	0.026
b <sub>2</sub> a E <sub>12</sub>	-0.0234	0.002	-13.8	7E-07	-0.027	-0.019
b <sub>3</sub> a E <sub>11</sub>	0.0014	4E-04	3.532	0.008	5E-04	0.002

(12.B)

Multiple R	0.989					
R Square	0.978					
Adjusted R Square	0.847					
Standard Error	5E-04					
	df	SS	MS	F	Significance F	
Regression	3	8E-05	3E-05	117	2E-06	
Residual	8	2E-06	2E-07			
Total	11	9E-05				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>2</sub>	0.0094	2E-04	40.62	1E-10	0.009	0.010
b <sub>3</sub> a E <sub>23</sub>	-0.0001	2E-04	-0.510	0.624	-0.001	4E-04
a <sub>2</sub> E <sub>1</sub>	0.0378	0.027	1.389	0.202	-0.025	0.100

(12.C)

Multiple R	0.999					
R Square	0.998					
Adjusted R Square	0.873					
Standard Error	0.001					
	df	SS	MS	F	Significance F	
Regression	3	0.007	0.002	1498	3E-10	
Residual	8	1E-05	2E-06			
Total	11	0.007				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>2</sub> a E <sub>32</sub>	-0.0076	0.001	-12.9	1E-06	-0.009	-0.006
b <sub>3</sub>	0.0114	0.001	18.6	7E-08	0.010	0.013
a <sub>3</sub> E <sub>1</sub>	0.9325	0.070	13.4	9E-07	0.772	1.093

b <sub>1</sub>	0.0230	b <sub>2</sub> a E <sub>32</sub>	0.0234	E12	2.5531
a <sub>1</sub>	0.9770	b <sub>3</sub> a E <sub>31</sub>	-0.0014	E13	-0.1274
b <sub>2</sub>	0.0094	a <sub>2</sub> E <sub>11</sub>	-0.0378	E21	-0.0381
a <sub>2</sub>	0.9906	b <sub>3</sub> a E <sub>33</sub>	0.0001	E23	0.0108
b <sub>3</sub>	0.0114	a <sub>3</sub> E <sub>11</sub>	-0.9325	E31	-0.9432
a <sub>3</sub>	0.9886	b <sub>2</sub> a E <sub>32</sub>	0.0076	E32	0.8211



5.

Y <sub>1</sub> Multiplier	0.15	P <sub>1</sub>	1.00
Y <sub>2</sub> Multiplier	1.00	P <sub>2</sub>	1.50
Y <sub>3</sub> Multiplier	0.15	P <sub>3</sub>	0.75

(12.A)

(12A)						
Multiple R	0.996					
R Square	0.992					
Adjusted R Square	0.865					
Standard Error	0.001					
	df	SS	MS	F	Significance F	
Regression	3	0.001	3E-04	330	7E-08	
Residual	8	7E-06	9E-07			
Total	11	0.001				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>1</sub>	0.0230	0.001	18.66	7E-08	0.020	0.026
b <sub>2</sub> a E <sub>12</sub>	-0.0351	0.003	-13.82	7E-07	-0.041	-0.029
b <sub>3</sub> a E <sub>13</sub>	0.0011	3E-04	3.532	0.008	4E-04	0.002

(12.B)

(12 B)

Multiple R	0.989
R Square	0.978
Adjusted R Square	0.847
Standard Error	5E-04

	df	SS	MS	F	Significance F
Regression	3	8E-05	3E-05	117	2E-06
Residual	8	2E-06	2E-07		
Total	11	9E-05			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>2</sub>	0.0141	3E-04	40.62	1E-10	0.013	0.015
b <sub>3</sub> a E <sub>2</sub>	-0.0001	2E-04	-0.510	0.624	-0.001	3E-04
a <sub>2</sub> E <sub>1</sub>	0.0378	0.027	1.389	0.202	-0.025	0.100

(12.C)

Multiple R	0.999					
R Square	0.998					
Adjusted R Square	0.873					
Standard Error	0.001					
	df	SS	MS	F	Significance F	
Regression	3	0.007	0.002	1498	3E-10	
Residual	8	1E-05	2E-06			
Total	11	0.007				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>2</sub> a E <sub>12</sub>	-0.0114	0.001	-12.88	1E-06	-0.013	-0.009
b <sub>3</sub>	0.0085	5E-04	18.63	7E-08	0.007	0.010
a <sub>3</sub> E <sub>1</sub>	0.9325	0.070	13.39	9E-07	0.772	1.093

b <sub>1</sub>	0.0230	b <sub>2</sub> a E <sub>12</sub>	0.0351	E12	2.5531
a <sub>1</sub>	0.9770	b <sub>3</sub> a E <sub>13</sub>	-0.0011	E13	-0.1274
b <sub>2</sub>	0.0141	a <sub>2</sub> E <sub>1</sub>	-0.0378	E21	-0.0383
a <sub>2</sub>	0.9859	b <sub>3</sub> a E <sub>13</sub>	0.0001	E23	0.0108
b <sub>3</sub>	0.0085	a <sub>1</sub> E <sub>1</sub>	-0.9325	E31	-0.9405
a <sub>3</sub>	0.9915	b <sub>2</sub> a E <sub>12</sub>	0.0114	E32	0.8188

6.

Y <sub>1</sub> Multiplier	0.15	P <sub>1</sub>	1.00
Y <sub>2</sub> Multiplier	1.00	P <sub>2</sub>	2.00
Y <sub>3</sub> Multiplier	0.15	P <sub>3</sub>	0.50

(12.A)

(12A)						
Multiple R	0.996					
R Square	0.992					
Adjusted R Square	0.865					
Standard Error	0.001					
	df	SS	MS	F	Significance F	
Regression	3	0.001	3E-04	330	7E-08	
Residual	8	7E-06	9E-07			
Total	11	0.001				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>1</sub>	0.0230	0.001	18.66	7E-08	0.020	0.026
b <sub>2</sub> a E <sub>12</sub>	-0.0468	0.003	-13.82	7E-07	-0.055	-0.039
b <sub>3</sub> a E <sub>13</sub>	0.0007	2E-04	3.532	0.008	2E-04	0.001

(12.B)

Multiple R	0.989					
R Square	0.978					
Adjusted R Square	0.847					
Standard Error	5E-04					
	df	SS	MS	F	Significance F	
Regression	3	8E-05	3E-05	117	2E-06	
Residual	8	2E-06	2E-07			
Total	11	9E-05				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>2</sub>	0.0187	5E-04	40.62	1E-10	0.018	0.020
b <sub>3</sub> a E <sub>23</sub>	-0.0001	1E-04	-0.510	0.624	-3E-04	2E-04
a <sub>2</sub> E <sub>1</sub>	0.0378	0.027	1.389	0.202	-0.025	0.100

(12.C)

Multiple R	0.999					
R Square	0.998					
Adjusted R Square	0.873					
Standard Error	0.001					
	df	SS	MS	F	Significance F	
Regression	3	0.007	0.002	1498	3E-10	
Residual	8	1E-05	2E-06			
Total	11	0.007				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>2</sub> a E <sub>22</sub>	-0.0152	0.001	-12.88	1E-06	-0.018	-0.012
b <sub>3</sub>	0.0057	3E-04	18.63	7E-08	0.005	0.006
a <sub>2</sub> E <sub>1</sub>	0.9325	0.070	13.39	9E-07	0.772	1.093

b <sub>1</sub>	0.0230	b <sub>1</sub> a E <sub>1</sub>	0.0468	E12	2.5531
a <sub>1</sub>	0.9770	b <sub>2</sub> a E <sub>1</sub>	-0.0007	E13	-0.1274
b <sub>2</sub>	0.0187	a <sub>2</sub> E <sub>1</sub>	-0.0378	E21	-0.0385
a <sub>1</sub>	0.9813	b <sub>3</sub> a E <sub>1</sub>	0.0001	E23	0.0109
b <sub>3</sub>	0.0057	a <sub>3</sub> E <sub>1</sub>	-0.9325	E31	-0.9378
a <sub>1</sub>	0.9943	b <sub>3</sub> a E <sub>2</sub>	0.0152	E32	0.8164

7.

Y <sub>1</sub> Multiplier	0.15	P <sub>1</sub>	1.00
Y <sub>2</sub> Multiplier	1.00	P <sub>2</sub>	0.50
Y <sub>3</sub> Multiplier	0.15	P <sub>3</sub>	2.00

(12.A)

(12.A)						
Multiple R	0.996					
R Square	0.992					
Adjusted R Square	0.865					
Standard Error	0.001					
	df	SS	MS	F	Significance F	
Regression	3	0.001	3E-04	330	7E-08	
Residual	8	7E-06	9E-07			
Total	11	0.001				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>1</sub>	0.0230	0.001	18.66	7E-08	0.020	0.026
b <sub>2</sub> a E <sub>12</sub>	-0.0117	0.001	-13.82	7E-07	-0.014	-0.010
b <sub>3</sub> a E <sub>13</sub>	0.0028	0.001	3.532	0.008	0.001	0.005

(12.B)

(12.5)

Multiple R	0.989
R Square	0.978
Adjusted R Square	0.847
Standard Error	5E-04

	df	SS	MS	F	Significance F
Regression	3	8E-05	3E-05	117	2E-06
Residual	8	2E-06	2E-07		
Total	11	9E-05			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>1</sub>	0.0047	1E-04	40.62	1E-10	0.004	0.005
b <sub>2</sub> a E <sub>12</sub>	-0.0002	5E-04	-0.510	0.624	-0.001	0.001
a <sub>2</sub> E <sub>1</sub>	0.0378	0.027	1.389	0.202	-0.025	0.100

(12.C)

12.07

Multiple R	0.999
R Square	0.998
Adjusted R Square	0.873
Standard Error	0.001

	df	SS	MS	F	Significance F
Regression	3	0.007	0.002	1498	3E-10
Residual	8	1E-05	2E-06		
Total	11	0.007			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
b <sub>2</sub> a E <sub>12</sub>	-0.0038	3E-04	-12.88	1E-06	-0.004	-0.003
b <sub>3</sub>	0.0227	0.001	18.63	7E-08	0.020	0.026
a <sub>2</sub> E <sub>1</sub>	0.9325	0.070	13.39	9E-07	0.772	1.093

b <sub>1</sub>	0.0230	b <sub>2</sub> a E <sub>12</sub>	0.0117	E12	2.5531
a <sub>1</sub>	0.9770	b <sub>3</sub> a E <sub>13</sub>	-0.0028	E13	-0.1274
b <sub>2</sub>	0.0047	a <sub>2</sub> E <sub>1</sub>	-0.0378	E21	-0.0379
a <sub>2</sub>	0.9953	b <sub>3</sub> a E <sub>13</sub>	0.0002	E23	0.0107
b <sub>3</sub>	0.0227	a <sub>2</sub> E <sub>1</sub>	-0.9325	E31	-0.9542
a <sub>3</sub>	0.9773	b <sub>3</sub> a E <sub>13</sub>	0.0038	E32	0.8307



## LIST OF REFERENCES

1. Terasawa, K.L. and Gates, W.R., *Allies, Adversaries And Commitment In Defense Alliances* : Working Paper No. 90-06, Naval Postgraduate School, Monterey, CA, 1990.
2. Jones, L.R. and Bixler, G.C., *Mission Financing To Realign National Defense: : Research In Public Policy Analysis And Management, Volume 5*, pp. 193-208, Jai Press, Inc., London, U.K., 1992.
3. Bush, George, *National Security Strategy of the United States*, The White House, 1991.
4. Jones, L.R. and Terasawa, K.L., *United States - Japan Economic, Trade and Security Relations*, Naval Postgraduate School, Monterey, CA, 1994.
5. Secretary of Defense, *Annual Report to the President and the Congress 1994*, The U.S. Department of Defense, 1993.
6. Kaufman, W.W., *Glasnost, Perestroika, and U.S. Defense Spending*, The Brookings Institution, Washington, D.C., 1990.
7. Pindyck, R.S. and Rubinfeld, D.L., *Microeconomics*, Second Edition, Macmillan Publishing, CO., 1992.
8. U.S. Arms Control and Disarmament Agency, *World Military Expenditures and Arms Transfers 1993 - 1994*, Washington, D.C., 1993.
9. Neter, J., Wasserman, W. and Whitmore, G.A., *Applied Statistics*, Fourth Edition, A Division of Simon and Schuster, Inc., MA, 1993.
10. Lancaster, K., *Mathematical Economics*, pp. 278-280, The macmillan Co, NY, 1968.
11. Hirsch, M.W. and Smale, S., *Differential Equations, Dynamical Systems, and Linear Algebra*, pp. 324-325, Academic Press, Inc, NY, 1974.
12. Japanese Defense Agency, *Defense of Japan 1994*, The Defense Agency, Japan, 1994.



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     Department of System Management  
     Naval Postgraduate School  
     Monterey, CA 93943-5103
  
5. Lieutenant Commander Yorihiro Nowada ..... 1  
     6-9-4 Ohwada-cho Hchihji-shi  
     Tokyo 192, Japan